Multivariable Control (ME-422) - Exercise session 7B

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In the previous exercise session, we introduced the Gripen system...

In this set of exercises, you will learn to design different control strategies for controlling the lateral dynamics of a JAS 39 Gripen aircraft flying at an altitude of 500 m with a speed of 730 $\frac{\text{km}}{\text{h}}$.

A folder containing all the necessary files for simulating the system is provided in Moodle.

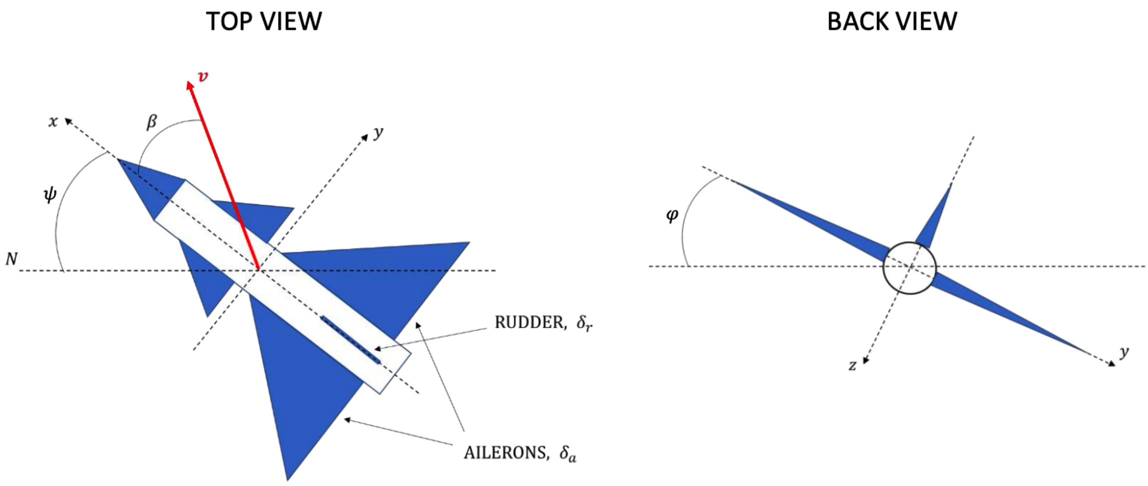

Figure 1: Top and back view of the Gripen aircraft

Linearized system model The continuous-time linearized dynamics of the aircraft are described by the state-space equations:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{1}$$

where $x \in \mathbb{R}^7$, $u \in \mathbb{R}^2$ and $y \in \mathbb{R}^2$. Table 1 summarizes the physical meaning of the different state coordinates, whereas Table 2 defines the control variables. The measured signals (outputs) are $y_1 = x_4 = \varphi$ and $y_2 = x_5 = \psi$. The values of the matrices A, B and C are defined in the `gripen_data.mat` file.

Assume that the system is initialized with:

$$\bar{v}_y = 10\,\frac{\text{m}}{\text{s}}\,, \quad \bar{p} = \bar{r} = \frac{\pi}{180}\,\frac{\text{rad}}{\text{s}}\,, \quad \bar{\varphi} = \frac{\pi}{36}\,\text{rad}\,, \quad \bar{\psi} = \frac{\pi}{18}\,\text{rad} \quad \text{and} \quad \bar{\delta_a} = \bar{\delta_r} = 0\,\text{rad}\,.$$

	Physical variable	Description (see Figure 1)	Units
x_1	v_y	$v_y \approx \beta v$ where v is the velocity	$\frac{\mathrm{m}}{\mathrm{s}}$
x_2	p	roll angular rate	$\frac{\mathrm{rad}}{\mathrm{s}}$
x_3	r	turning angular rate	$\frac{\mathrm{rad}}{\mathrm{s}}$
x_4	φ	roll angle	rad
x_5	ψ	course angle	rad
x_6	δ_a	aileron angle	rad
x_7	δ_r	rudder angle	rad

Table 1: States of the Gripen system

	Physical variable	Description (see Figure 1)	Units
u_1	δ_a^{cmd}	aileron command angle	rad
u_2	δ_r^{cmd}	rudder command angle	rad

Table 2: Control variables

... and you were asked to:

1. Load `gripen_data.mat` in Matlab to define the matrices of the linearized model of the Gripen.
2. Discretize the continuous time model with sampling time $T_s \in \{0.5, 0.05, 0.005\}$ using both the exact and the forward Euler discretization methods.
   (a) Is stability always preserved? Verify your answer by simulating both the continuous-time and the discrete-time models using Simulink. To do this, assume that $u_1 = u_2 = 0$.
   Hint: use the Simulink blocks (discrete) state space and zero-order hold.
3. Choose a suitable value for T_s by analyzing the poles of the continuous-time system.

In this exercise session, you are asked to:

From now on, consider the discrete time system obtained in point 2 using the exact discretization method and $T_s = 0.005\,\mathrm{s}$.

4. Assume that the states are measured.
   (a) Is the system reachable using both inputs? Design a state-feedback controller assigning the closed-loop eigenvalues in $e^{-0.1\,T_s}$, $e^{-0.1\,T_s}$, $e^{-1\,T_s}$, $e^{-2\,T_s}$, $e^{-3\,T_s}$, $e^{-5\,T_s}$ and $e^{-5\,T_s}$.
   (b) Design a new controller that makes the dynamics converge faster. Assign the eigenvalues in $e^{-1\,T_s}$, $e^{-2\,T_s}$, $e^{-3\,T_s}$, $e^{-4\,T_s}$, $e^{-5\,T_s}$, $e^{-20\,T_s}$ and $e^{-30\,T_s}$. Compare the input signals computed by the current controller with the ones in point 4a.
   (c) Is the system reachable using a single input? If possible, design a controller for assigning the closed-loop eigenvalues as in the point 4b using only the second input.
5. Assume that only the partial information given by $y(t)$ is available.
   (a) Design, if possible, a Luenberger observer in order to remove the assumption of fully measurable state of point 4. Plot the estimation error responses to validate the design.
      i. Assume now, that the system measurements are corrupted by constant disturbances of known magnitude $[\alpha_1, \alpha_2]^\top$. How is the asymptotic estimation error affected? Derive an expression of the steady state estimation error as a function of α_1, α_2. Then, simulate it for different numerical values of the constant disturbances.

(b) Since the roll and course angles (φ, ψ) are states that are always measurable, design, if possible, a reduced order observer for the remaining states.