Multivariable Control (ME-422) - Exercise session 6B

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In this set of exercises, you will learn to design different control strategies for controlling the lateral dynamics of a JAS 39 Gripen aircraft flying at an altitude of 500 m with a speed of 730 $\frac{\text{km}}{\text{h}}$.

A folder containing all the necessary files for simulating the system is provided in Moodle.

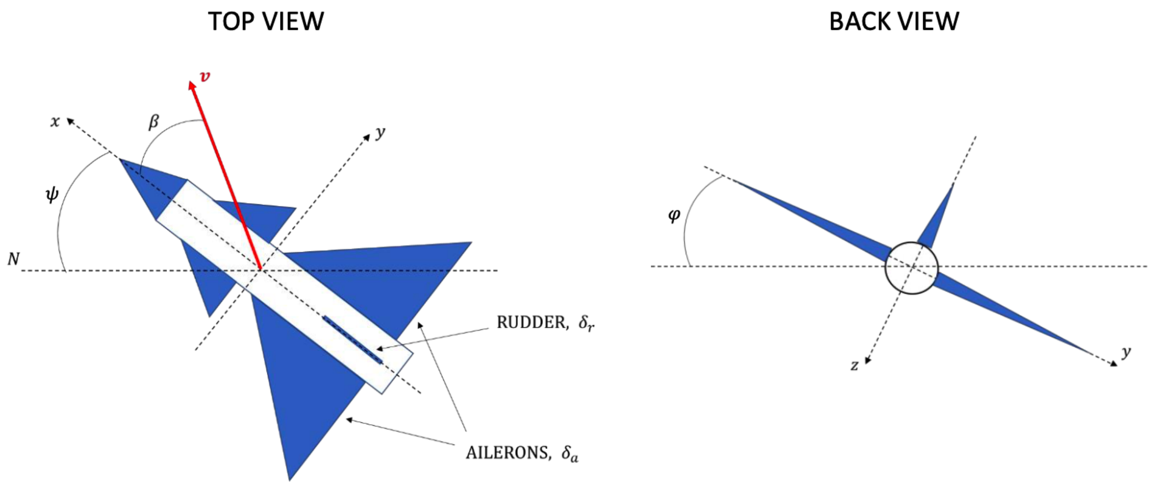

Figure 1: Top and back view of the Gripen aircraft

Linearized system model The continuous-time linearized dynamics of the aircraft are described by the state-space equations:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{1}$$

where $x \in \mathbb{R}^7$, $u \in \mathbb{R}^2$ and $y \in \mathbb{R}^2$. Table 1 summarizes the physical meaning of the different state coordinates, whereas Table 2 defines the control variables. The measured signals (outputs) are $y_1 = x_4 = \varphi$ and $y_2 = x_5 = \psi$. The values of the matrices A, B and C are defined in the `gripen_data.mat` file.

Assume that the system is initialized with:

$$\bar{v}_y = 10\,\frac{\text{m}}{\text{s}}\,, \quad \bar{p} = \bar{r} = \frac{\pi}{180}\,\frac{\text{rad}}{\text{s}}\,, \quad \bar{\varphi} = \frac{\pi}{36}\,\text{rad}\,, \quad \bar{\psi} = \frac{\pi}{18}\,\text{rad} \quad \text{and} \quad \bar{\delta_a} = \bar{\delta_r} = 0\,\text{rad}\,.$$

1. Load `gripen_data.mat` in Matlab to define the matrices of the linearized model of the Gripen.

	Physical variable	Description (see Figure 1)	Units
x_1	v_y	$v_y \approx \beta v$ where v is the velocity	$\frac{\mathrm{m}}{\mathrm{s}}$
x_2	p	roll angular rate	$\frac{\mathrm{rad}}{\mathrm{s}}$
x_3	r	turning angular rate	$\frac{\mathrm{rad}}{\mathrm{s}}$
x_4	φ	roll angle	rad
x_5	ψ	course angle	rad
x_6	δ_a	aileron angle	rad
x_7	δ_r	rudder angle	rad

Table 1: States of the Gripen system

	Physical variable	Description (see Figure 1)	Units
u_1	δ_a^{cmd}	aileron command angle	rad
u_2	δ_r^{cmd}	rudder command angle	rad

Table 2: Control variables

2. Discretize the continuous time model with sampling time $T_s \in \{0.5, 0.05, 0.005\}$ using both the exact and the forward Euler discretization methods.
   (a) Is stability always preserved? Verify your answer by simulating both the continuous-time and the discrete-time models using Simulink. To do this, assume that $u_1 = u_2 = 0$.
   Hint: use the Simulink blocks (discrete) state space and zero-order hold.
3. Choose a suitable value for T_s by analyzing the poles of the continuous-time system.