Lecture 7

Offset-free tracking
Disturbance estimation

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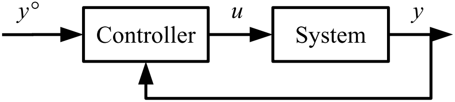
Motivations

- So far, we focused on regulation (AS of the origin) and performance through eigenvalues assignment (EA)
- So far, no disturbances!

In several applications one wants to design controllers

- for solving tracking problems, especially for constant references y^0



- for rejecting the effect of disturbances

Terminology

Offset-free tracking when :

- disturbances and setpoints are *constant*
- $\|y^o - y(t)\| \to 0$ as $t \to +\infty$, independently of the initial state $x(0)$ of the system

Summary

- Necessary properties of the system under control
- Two control design methods
  - Feedforward compensation (measurable disturbance)
  - Integral control (unknown disturbance)
- Disturbance estimation (unknown disturbance with known dynamics)

Problem setup

$$P: \begin{cases} x^{+} = Ax + Bu + Md \\ y = Cx + Nd \end{cases}$$

$x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$ and disturbance $d \in \mathbb{R}^r$

Flexible formulation: the disturbance can act on inputs/states/outputs

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Problem

Design an LTI controller for P such that

1. the closed-loop system is AS and has eigenvalues in prescribed positions
2. $\lim_{k \to +\infty} y(k) = y^o$ for constant y^o and d

Remark

1. $\Rightarrow$ stability and performance during transients
2. $\Rightarrow$ zero steady-state tracking error for constant y^o and d

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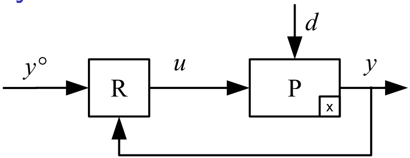
Standing assumptions

- (A, B) reachable and (A, C) observable
- **Square system:** $m = p$

Steady-state analysis



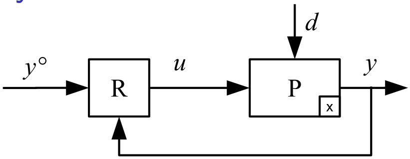
If the CL system is AS and point (2) is verified, then $x \to \bar{x}$ and $u \to \bar{u}$ for $t \to +\infty$, where $\bar{x}$ and $\bar{u}$ yield $y = y^o$

$$\bar{x} = A\bar{x} + B\bar{u} + Md$$
$$y^o = C\bar{x} + Nd$$

In matrix form, for arbitrary y^o and d,

$$\underbrace{\begin{bmatrix} A - I & B \\ C & 0 \end{bmatrix}}_{\Sigma} \begin{bmatrix} \bar{x} \\ \bar{u} \end{bmatrix} = \begin{bmatrix} 0 & -M \\ I & -N \end{bmatrix} \begin{bmatrix} y^o \\ d \end{bmatrix} \qquad (*)$$

Steady-state analysis



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Problem

Can the condition $(*)$ be fulfilled by any system verifying the standing assumptions? NO

Necessary steady-state condition for offset-free tracking

$$\underbrace{\begin{bmatrix} A-I & B \\ C & 0 \end{bmatrix}}_{\Sigma} \begin{bmatrix} \bar{x} \\ \bar{u} \end{bmatrix} = \begin{bmatrix} 0 & -M \\ I & -N \end{bmatrix} \begin{bmatrix} y^o \\ d \end{bmatrix} \qquad (*)$$

Proposition

Assume (A, B) is reachable. Then, $(*)$ has a unique solution $\begin{bmatrix} \bar{x}^T & \bar{u}^T \end{bmatrix}^T$ for any $\begin{bmatrix} y^{o\,T} & d^T \end{bmatrix}^T \in \mathbb{R}^{m+r}$ if and only if $\det(\Sigma) \neq 0$

Steady-state condition $\det(\Sigma) \neq 0$

- $\det(\Sigma) \neq 0$ is a property of the system only. If not verified ,tracking of arbitrary setpoints cannot be guaranteed, whatever controller is used
- Unique equilibrium $(\bar{x}, \bar{u})$ given by

$$\begin{bmatrix} \bar{x} \\ \bar{u} \end{bmatrix} = \Sigma^{-1} \begin{bmatrix} 0 & -M \\ I & -N \end{bmatrix} \begin{bmatrix} y^o \\ d \end{bmatrix} \quad \Sigma = \begin{bmatrix} A - I & B \\ C & 0 \end{bmatrix}$$

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Definition

The transmission zeros of a MIMO LTI system are the values $z \in \mathbb{C}$ such that

$$\det\left(\begin{bmatrix} A - zI & B \\ C & 0 \end{bmatrix}\right) = 0 \qquad (*)$$

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Motivation for the terminology

One can show that for a SISO system, a transmission zero is a zero of the transfer function $G(z) = C(zI - A)^{-1}B$

Steady-state condition $\det(\Sigma) \neq 0$

$$\det\left(\begin{bmatrix} A-I & B \\ C & 0 \end{bmatrix}\right) \neq 0 \qquad (*)$$

$(*)$ means that the system has no "discrete derivators" (zeros equal to $+1$).

- In the SISO case, the presence of a derivator means that the gain of the system is zero, which implies that, for constant input, the only constant output in steady state is zero.
  - Consider, for instance $G(z) = \frac{z-1}{z+0.5} = \frac{Y(z)}{U(z)}$

$$\begin{aligned} U(z)(z-1) &= Y(z)(z+0.5) \\ &\downarrow z = \text{push forward in time} \\ u(k+1) - u(k) &= y(k+1) + 0.5y(k) \\ &\downarrow \text{ in steady state, assuming } y(k) = \bar{y} \text{ constant} \\ 0 &= 1.5\bar{y} \end{aligned}$$

Mid-lecture summary

We considered a square, reachable and observable system. The original problem we wanted to solve is

Design an LTI controller for P such that

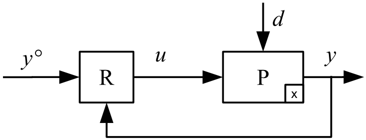
1. the closed-loop system is AS and has eigenvalues in prescribed positions
2. $\lim_{k \to +\infty} y(k) = y^o$ for constant y^o and d

A necessary condition is

$$\det(\Sigma) \neq 0, \quad \Sigma = \begin{bmatrix} A - I & B \\ C & 0 \end{bmatrix} \qquad (*)$$

Two design approaches to offset-free tracking

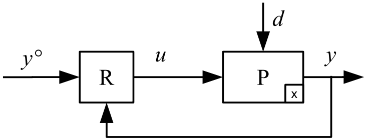
Method 1 : Feedforward compensation



Assumption

$d \in \mathbb{R}^r$ is measured

Method 1 : Feedforward compensation



Assumption

$d \in \mathbb{R}^r$ is measured

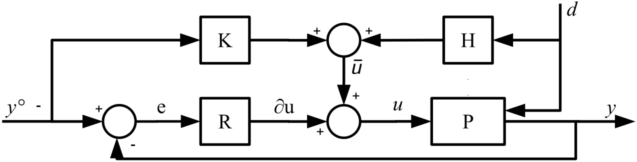
Step 1 : find $\bar{u}$ in steady state as a function of y^o and d

$$\begin{bmatrix} \bar{x} \\ \bar{u} \end{bmatrix} = \Sigma^{-1} \begin{bmatrix} 0 & -M \\ I & -N \end{bmatrix} \begin{bmatrix} y^o \\ d \end{bmatrix}$$

$$\bar{u} = \begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{u} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & I \end{bmatrix} \Sigma^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix}}_{K} y^o + \underbrace{\begin{bmatrix} 0 & I \end{bmatrix} \Sigma^{-1} \begin{bmatrix} -M \\ -N \end{bmatrix}}_{H} d$$

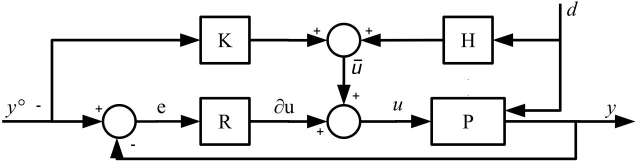
$$\bar{u} = K y^o + H d$$

Controller with feedforward compensation



The architecture assumes that the controller is driven by the tracking error

Controller with feedforward compensation



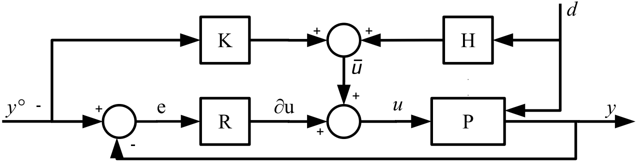
The architecture assumes that the controller is driven by the tracking error

Set y^o and d to zero and assume that $\det(\Sigma) \neq 0$ and that the closed-loop system is AS. Then, for constant inputs y^o and d, there is a single set of steady-state variables (states/inputs/outputs of R and P).

- In particular, $e \to \bar{e} + 0$. Then $\partial u \to 0$ and

$$\partial u \to 0 \Rightarrow u \to \bar{u} \Rightarrow y \to y^o$$

Controller with feedforward compensation



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$$\partial u \to 0 \Rightarrow u \to \bar{u} \Rightarrow y \to y^o$$

Design of the controller R

Must guarantee AS and assign closed-loop eigenvalues as desired

Key conditions: the system "seen" from R (meaning with inputs ∂u, d and y^o and output e) must be reachable and observable

Dynamics of system $(\partial u, d, y^o) \rightarrow e$

Set $x(k) = \bar{x} + \partial x(k), \quad u(k) = \bar{u} + \partial u(k), \quad y(k) = y^o - e(k)$.
From the LTI dynamics of P

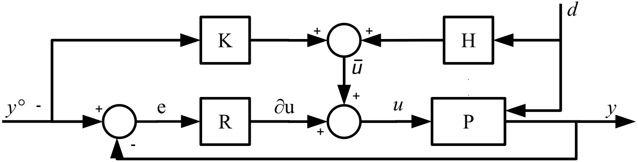
$$
\begin{aligned}
\bar{x} + \partial x(k+1) &= A\bar{x} + A\partial x(k) + B\bar{u} + B\partial u(k) + Md \\
y^o - e(k) &= C\bar{x} + C\partial x(k) + Nd
\end{aligned}
$$

Recalling that $\bar{x} = A\bar{x} + B\bar{u} + Md$ and $y^o = C\bar{x} + Nd$, we have

$$
\partial P : \begin{cases} \partial x(k+1) = A\partial x(k) + B\partial u(k) \\ e(k) = -C\partial x(k) \end{cases}
$$

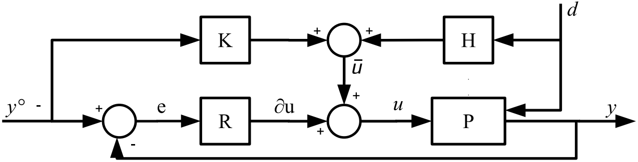
- By construction, ∂P is independent of y^o and d
  - For control design only ∂P matters and one can set $d = 0$ and $y^o = 0$
- ∂P is reachable and observable if the system P has these properties

Design of the controller R



- Use eigenvalues assignment + observer for designing R

Design of the controller R



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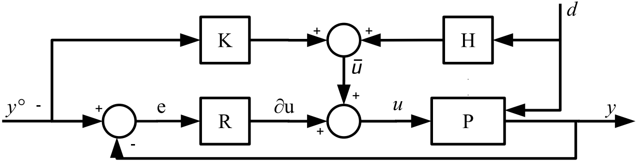
Pros of the approach

- Simplicity
- K, H are static compensators (easy to implement)

Design of the controller R



- Use eigenvalues assignment + observer for designing R

Pros of the approach

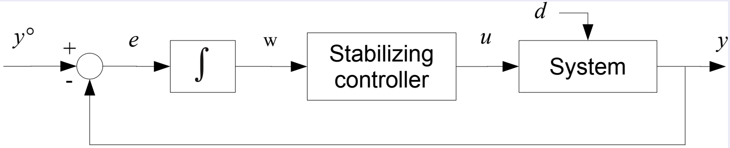
- Simplicity
- K, H are static compensators (easy to implement)

Cons of the approach

- Tracking and disturbance rejection are not robust against uncertainties in the parameters of P
  - If some entries of (A, B, C, D) differ from their nominal values, $e \to \bar{e} \neq 0$ as $t \to +\infty$
- The disturbance must be measured

Method II: controller with integrators

Lessons from basic control theory for SISO CT systems



Step 1: design of the static part of the controller

- "add an integrator to drive the error to zero when the setpoint is constant"

Method II: controller with integrators

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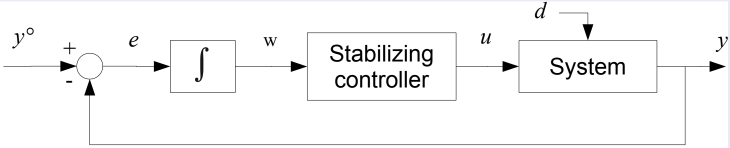


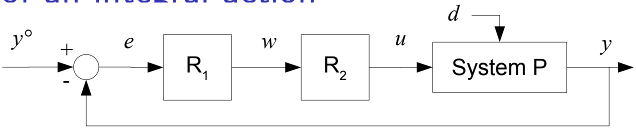
Step 1: design of the static part of the controller

- "add an integrator to drive the error to zero when the setpoint is constant"

Step 2: design of the dynamic part of the regulator through loopshaping. This design step considers the cascade of the system under control and the integrator

- Provides offset-free tracking even if $d(t)$ is unknown (but constant)

Addition of an integral action



- $d \in \mathbb{R}^n$ unknown and constant, controller driven by the tracking error

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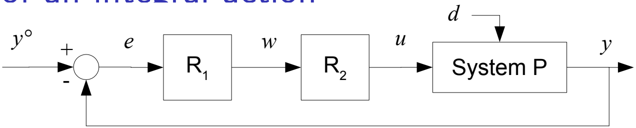

- $d \in \mathbb{R}^n$ unknown and constant, controller driven by the tracking error

Step 1 - Design of R_1: p integrators

$$R_1 : \begin{cases} v(k+1) = v(k) + e(k) & v(k) \in \mathbb{R}^p \\ w(k) = v(k) \end{cases}$$

Assume the closed-loop system is AS and $\det(\Sigma) \neq 0$. Then, for d and y^o constant, the system reaches a steady state. The steady state values $\bar{v}$, $\bar{w}$, $\bar{e}$, of $v(k)$, $w(k)$ and $e(k)$ verify

$$\begin{cases} \bar{v} = \bar{v} + \bar{e} \\ \bar{w} = \bar{v} \end{cases} \Rightarrow \bar{e} = 0$$

Offset-free tracking achieved!

Design of R_2

- **Goal:** R_2 must stabilize the closed-loop system
- **Key conditions:** the system "seen" from R_2 (i.e. with inputs u, d , and y^o and output w) must be reachable and observable and the plant must verify $\det(\Sigma) \neq 0$

Dynamics of system $(u, d, y^0) \to w$

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Md \\ v(k+1) = v(k) + y^o - Cx(k) - Nd \\ w(k) = v(k) \end{cases}$$

Setting $\eta = \left[x^T, v^T\right]^T$

$$\eta^+ = \underbrace{\begin{bmatrix} A & 0 \\ -C & I \end{bmatrix}}_{\bar{A}} \eta + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{\bar{B}} u + \underbrace{\begin{bmatrix} M \\ -N \end{bmatrix}}_{\bar{M}} d + \begin{bmatrix} 0 \\ I \end{bmatrix} y^o$$

$$w = \underbrace{\begin{bmatrix} 0 & I \end{bmatrix}}_{\bar{C}} \eta + \underbrace{0}_{\bar{N}} d$$

Remark: for control design, only the matrices $(\bar{A}, \bar{B}, \bar{C})$ matter $\to$ one can set $y^o = 0$ and $d = 0$

Observability of $(\bar{A}, \bar{B}, \bar{C})$

The observability matrix of $(\bar{A}, \bar{C})$ is

$$M_0 = \begin{bmatrix} 0 & -C^T & -A^T C^T - C^T & -\left(A^2\right)^T C^T - A^T C^T - C^T & \cdots \\ I & I & I & I & \cdots \end{bmatrix}$$

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$$M_0 = \left[\begin{array}{cccccc} 0 & -C^T & -A^T C^T - C^T & -\left(A^2\right)^T C^T - A^T C^T - C^T & \cdots \\ I & I & I & I & \cdots \end{array} \right]$$

And, replacing (column i) with [-(column i)+(column $i-1$)], for $i > 1$

$$\tilde{M}_0 = \left[\begin{array}{ccccc} 0 & C^T & A^T C^T & \left(A^2\right)^T C^T & \cdots \\ I & 0 & 0 & 0 & \cdots \end{array} \right]$$

Observability of $(\bar{A}, \bar{B}, \bar{C})$

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$$\tilde{M}_0 = \begin{bmatrix} 0 & C^T & A^TC^T & \left(A^2\right)^T C^T & \cdots \\ I & 0 & 0 & 0 & \cdots \end{bmatrix}$$

Lemma

M_0 has maximal rank if (A, C) is observable.

Proof : The upper-right block of M_0^T is the observability matrix of (A, C)

Reachability of $(\bar{A}, \bar{B}, \bar{C})$

The reachability matrix of $(\bar{A}, \bar{B})$ is

$$M_r = \begin{bmatrix} B & AB & A^2B & A^3B & \cdots \\ 0 & -CB & -CAB - CB & -CA^2B - CAB - CB & \cdots \end{bmatrix}$$

Reachability of $(\bar{A}, \bar{B}, \bar{C})$

The reachability matrix of $(\bar{A}, \bar{B})$ is

$$M_r = \begin{bmatrix} B & AB & A^2B & A^3B & \cdots \\ 0 & -CB & -CAB - CB & -CA^2B - CAB - CB & \cdots \end{bmatrix}$$

Lemma

M_r has maximal rank if (A, B) is reachable and $\det(\Sigma) \neq 0$,

$$\Sigma = \begin{bmatrix} A - I & B \\ C & 0 \end{bmatrix}$$

Proof : omitted...

Controller with integrator : pros and cons

Pros : If entries of (A, B, C, D) differ from their nominal value, often their steady-state effect can be "dumped" into $d(k)$

- The controller is robust against these perturbations

Controller with integrator : pros and cons

Pros : If entries of (A, B, C, D) differ from their nominal value, often their steady-state effect can be "dumped" into $d(k)$

- The controller is robust against these perturbations

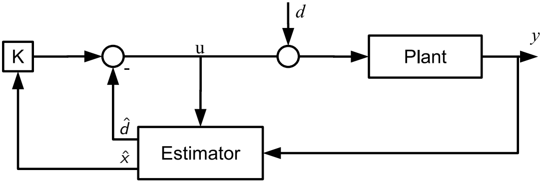
Cons : d and y^o must be constant for perfect tracking. What about non constant $d(k)$ and $y^o(k)$?

Disturbance estimation

Disturbance estimation

Idea for non constant disturbance: estimate and subtract.
We consider only the problem of guaranteeing that the plant state $x(k)$ converges to zero as $k \to +\infty$ (not a tracking problem)

Reference diagram



Remarks

$d(k)$ is assumed to affect the control variable

- If the disturbance acts elsewhere, it has to be brought in this position ("virtual disturbance")
- The estimator provides both $\hat{d}$ and $\hat{x}$ but only $\hat{x}$ is used by the state-feedback controller K

Problem setup

Plant

$$x^+ = Ax + B(u + d)$$
$$y = Cx$$

Disturbance generator

$$x_d^+ = \phi x_d$$
$$d = Hx_d$$

Disturbance model (ϕ, H)

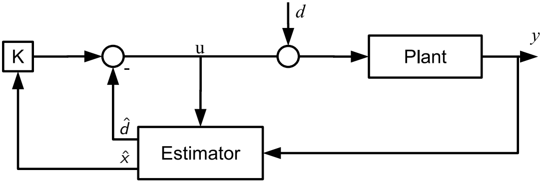
- Needed for state estimation
- The initial state is unknown
- LTI models are quite flexible. For instance they embrace

$$\text{constant disturbances} \begin{cases} x_d^+ = x_d \\ d = x_d \end{cases}$$

$$\underset{\text{(unstable model)}}{\text{ramp disturbances}} \begin{cases} x_{d,1}^+ = x_{d,2} + x_{d,1} \\ x_{d,2}^+ = x_{d,2} \\ d = x_{d,1} \end{cases}$$

... and any disturbance that can be represented as the free state of an LTI system (recall the lectures on modes!)

Design of observer/controller



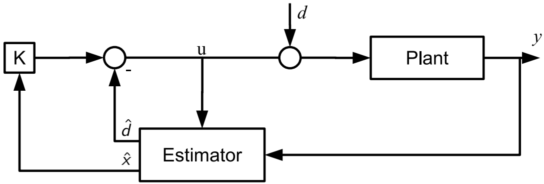
Augmented system (plant + disturbance generator)

$$\begin{bmatrix} x^+ \\ x_d^+ \end{bmatrix} = \underbrace{\begin{bmatrix} A & BH \\ 0 & \phi \end{bmatrix}}_{\bar{A}} \begin{bmatrix} x \\ x_d \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{\bar{B}} u$$

$$y = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{\bar{C}} \begin{bmatrix} x \\ x_d \end{bmatrix}$$

Important remark : $(\bar{A}, \bar{B}, \bar{C})$ is always uncontrollable because the disturbance model is a free system!

Design of observer/controller



Design procedure

Assumptions:

- $\det(\Sigma) \neq 0$
- (A, B) reachable
- $(\bar{A}, \bar{C})$ observable

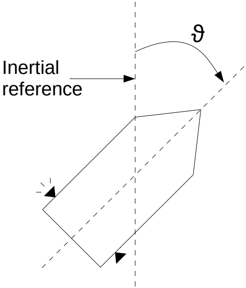
Algorithm:

- design K based on (A, B) only
- design the observer based on $(\bar{A}, \bar{C})$ for producing an AS error dynamics

The proof of stability of the closed-loop system is omitted

Example: disturbance torque rejection for a spinning satellite

Attitude control = proper orientation of the satellite antenna with respect to earth.



$$I\ddot{\theta} = M_C + M_D$$

$I =$ moment of inertia of the satellite (about the mass center)

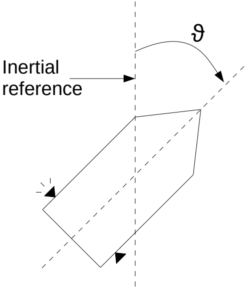
$M_C =$ control torque applied by thrusters

$M_D =$ disturbance torque

$\theta =$ angle of satellite

Example: disturbance torque rejection for a spinning satellite

Attitude control = proper orientation of the satellite antenna with respect to earth.



- Model in normalized coordinates :

$$u = \frac{M_C}{I}, \quad d = \frac{M_D}{I}$$

$$\ddot{\theta} = u + d$$

DT model ($x_1 = \theta$, $x_2 = \dot{\theta}$, $y = \theta$, exact discretisation)

As seen in lecture 5

$$\begin{bmatrix} x_1^+ \\ x_2^+ \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix}}_{B} (u + d)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

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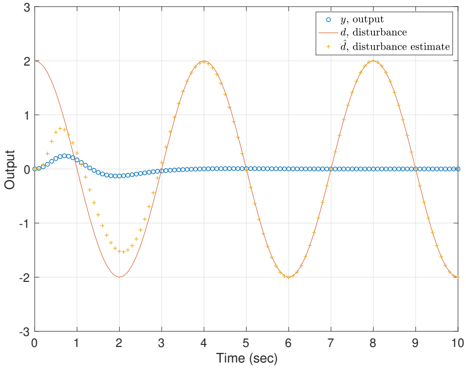
The disturbance torque $d(k)$ is generated by a solar pressure of $2\frac{\text{deg}}{\text{sec}^2}$ and acts is a sinusoid given by

$$\begin{bmatrix} x_{d,1}^+ \\ x_{d,2}^+ \end{bmatrix} = \begin{bmatrix} 0.9877 & 0.0996 \\ -0.2457 & 0.9877 \end{bmatrix} \begin{bmatrix} x_{d1} \\ x_{d2} \end{bmatrix}$$

$$d = x_{d1}$$

- Use the controller $K = [-10.2500, -3.4875]$, seen previously that places the control eigenvalues in $0.8 \pm j0.25$
- Place the observer eigenvalues at $0.4 \pm j0.4$, $0.9 \pm j0.1$

Simulations



- By using eigenvalue assignment, the gain of the full-order observer is

$$L = \begin{bmatrix} -1.3754 & -6.8124 & -8.0547 & 7.3964 \end{bmatrix}^T$$

Concluding remarks

- Methods for solving the regulation problem (AS of the origin) can be applied in a smart way for addressing

Concluding remarks

- Methods for solving the regulation problem (AS of the origin) can be applied in a smart way for addressing
  - offset-free tracking
  - disturbance estimation and rejection
- Similar design procedures exist for CT systems
- Generalisation for tracking non-constant references $y^o(k)$ exist
  - Key assumption : $y^o(k)$ is the output of an exogenous and autonomous system (A^o, C^o)