Multivariable Control (ME-422) - Exercise session 14B

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In the previous exercise sessions, we introduced the Gripen system...

In this set of exercises, you will learn to design different control strategies for controlling the lateral dynamics of a JAS 39 Gripen aircraft flying at an altitude of 500 m with a speed of 730 $\frac{\text{km}}{\text{h}}$.

A folder containing all the necessary files for simulating the system is provided in Moodle.

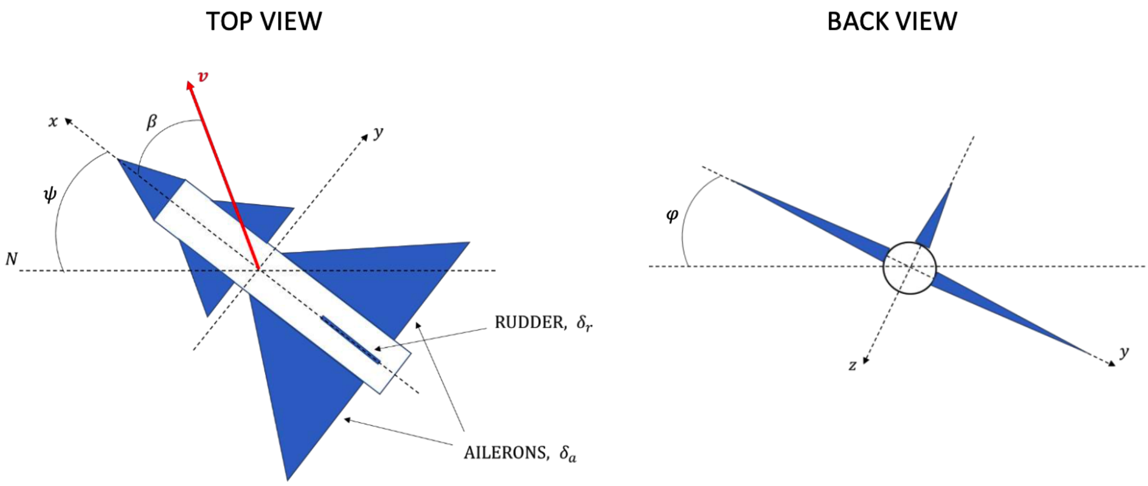

Figure 1: Top and back view of the Gripen aircraft

Linearized system model The continuous-time linearized dynamics of the aircraft are described by the state-space equations:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{1}$$

where $x \in \mathbb{R}^7$, $u \in \mathbb{R}^2$ and $y \in \mathbb{R}^2$. Table 1 summarizes the physical meaning of the different state coordinates, whereas Table 2 defines the control variables. The measured signals (outputs) are $y_1 = x_4 = \varphi$ and $y_2 = x_5 = \psi$. The values of the matrices A, B and C are defined in the `gripen_data.mat` file.

Assume that the system is initialized with:

$$\bar{v}_y = 10\,\frac{\text{m}}{\text{s}}\,, \quad \bar{p} = \bar{r} = \frac{\pi}{180}\,\frac{\text{rad}}{\text{s}}\,, \quad \bar{\varphi} = \frac{\pi}{36}\,\text{rad}\,, \quad \bar{\psi} = \frac{\pi}{18}\,\text{rad} \quad \text{and} \quad \bar{\delta_a} = \bar{\delta_r} = 0\,\text{rad}\,.$$

	Physical variable	Description (see Figure 2)	Units
x_1	v_y	$v_y \approx \beta v$ where v is the velocity	$\frac{\mathrm{m}}{\mathrm{s}}$
x_2	p	roll angular rate	$\frac{\mathrm{rad}}{\mathrm{s}}$
x_3	r	turning angular rate	$\frac{\mathrm{rad}}{\mathrm{s}}$
x_4	φ	roll angle	rad
x_5	ψ	course angle	rad
x_6	δ_a	aileron angle	rad
x_7	δ_r	rudder angle	rad

Table 1: States of the Gripen system

	Physical variable	Description (see Figure 2)	Units
u_1	δ_a^{cmd}	aileron command angle	rad
u_2	δ_r^{cmd}	rudder command angle	rad

Table 2: Control variables

... and you were asked to:

1. Load `gripen_data.mat` in Matlab to define the matrices of the linearized model of the Gripen.
2. Discretize the continuous time model with sampling time $T_s \in \{0.5, 0.05, 0.005\}$ using both the exact and the forward Euler discretization methods.
   (a) Is stability always preserved? Verify your answer by simulating both the continuous-time and the discrete-time models using Simulink. To do this, assume that $u_1 = u_2 = 0$.
   Hint: use the Simulink blocks (discrete) state space and zero-order hold.
3. Choose a suitable value for T_s by analyzing the poles of the continuous-time system.

From now on, consider the discrete time system obtained in point 2 using the exact discretization method and $T_s = 0.005\,\mathrm{s}$.

4. Assume that the states are measured.
   (a) Is the system reachable using both inputs? Design a state-feedback controller assigning the closed-loop eigenvalues in $e^{-0.1\,T_s}$, $e^{-0.1\,T_s}$, $e^{-1\,T_s}$, $e^{-2\,T_s}$, $e^{-3\,T_s}$, $e^{-5\,T_s}$ and $e^{-5\,T_s}$.
   (b) Design a new controller that makes the dynamics converge faster. Assign the eigenvalues in $e^{-1\,T_s}$, $e^{-2\,T_s}$, $e^{-3\,T_s}$, $e^{-4\,T_s}$, $e^{-5\,T_s}$, $e^{-20\,T_s}$ and $e^{-30\,T_s}$. Compare the input signals computed by the current controller with the ones in point 4a.
   (c) Is the system reachable using a single input? If possible, design a controller for assigning the closed-loop eigenvalues as in the point 4b using only the second input.
5. Assume that only the partial information given by $y(t)$ is available.
   (a) Design, if possible, a Luenberger observer in order to remove the assumption of fully measurable state of point 4. Plot the estimation error responses to validate the design.
      i. Assume now, that the system measurements are corrupted by constant disturbances of magnitude $[\alpha_1, \alpha_2]^\top$. How is the asymptotic estimation error affected? Derive an expression of the steady state estimation error as a function of α_1, α_2. Then, simulate it for different numerical values of the constant disturbances.

(b) Since the roll and course angles (φ, ψ) are states that are always measurable, design, if possible, a reduced order observer for the remaining states.

6. Assume an unknown but constant disturbance acts on the plant states. Design a controller with integral action to achieve asymptotic perfect tracking of a constant output reference. Implement and validate the designed controller architecture in Simulink. For the controller design, solve an eigenvalue assignment problem using pole placement.
Hint: consider augmenting the system with a chain of integrators and then design a compensator to stabilize the augmented closed-loop system. You can use the same observer you designed in the previous exercise session to estimate the system states from the measurements of the output.

7. Design a state-feedback controller by solving an infinite-horizon linear quadratic optimal control problem. Use the normalization approach to set the values of the weight matrices Q and R. To do so, assume that:

$$\begin{gathered}|x_1| \leq \bar{x}_1 = 12\,\frac{\mathrm{m}}{\mathrm{s}}\,, \quad |x_2| \leq \bar{x}_2 = \frac{5\pi}{36}\,\frac{\mathrm{rad}}{\mathrm{s}}\,, \quad |x_3| \leq \bar{x}_3 = \frac{5\pi}{36}\,\frac{\mathrm{rad}}{\mathrm{s}}\,, \quad |x_4| \leq \bar{x}_4 = \frac{\pi}{3}\,\frac{\mathrm{rad}}{\mathrm{s}}\,, \\ |x_5| \leq \bar{x}_5 = \frac{\pi}{3}\,\frac{\mathrm{rad}}{\mathrm{s}}\,, \quad |x_6| \leq \bar{x}_6 = \frac{13\pi}{36}\,\frac{\mathrm{rad}}{\mathrm{s}}\,, \quad |x_7| \leq \bar{x}_7 = \frac{13\pi}{36}\,\frac{\mathrm{rad}}{\mathrm{s}}\,, \\ |u_1| \leq \frac{\pi}{6}\,\frac{\mathrm{rad}}{\mathrm{s}} \quad \text{and} \quad |u_2| \leq \frac{\pi}{6}\,\frac{\mathrm{rad}}{\mathrm{s}}\,. \end{gathered} \tag{2}$$

Verify that the optimal feedback gain stabilizes the closed-loop system.

8. Assume now that the evolution of the system states and the measurement of the system outputs are corrupted by independent and identically distributed Gaussian disturbances.

   (a) Design a steady-state Kalman predictor to reconstruct the state with minimum error variance. Compare the state estimates produced by the Kalman filter with those provided by the Luenberger observer designed in item 5. For simplicity, assume that V and W are diagonal with $V_{1,1} = V_{2,2} = 0.0001$ and $W_{i,i} = 0.0001\bar{x}_i$ for $i = 1, \ldots, 7$, where $\bar{x}_i$ are given in (2).

   (b) Compute the mean of the innovation sequence $\nu_k = y_k - C\hat{x}_{k|k-1}$, and verify that it is close to zero as there is no mismatch between the real and assumed models.

   (c) Simulate the closed-loop system obtained by combining the Kalman predictor with the linear quadratic controller designed in the previous points.

Exercise session 14: optimal distributed control

9. The ground speed v for the Gripen is obtained by firing a radar beam towards the ground and measuring the Doppler shift of the returning beam. This ground speed measurement is essential in correctly controlling the lateral velocity v_y.

   However, the sensor has become unavailable due to a cyber-attack, and you have to make do with the other available state measurements. Your controller must now comply with a sparsity pattern

$$S = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

   In order to cope with the fact that v_y cannot be measured anymore, it is worth increasing the cost weight $Q(1,1)$ by 10 times.

   a) What is the new LQR controller K_{LQR}? Does it comply with the desired sparsity pattern?

   The first idea is to just set the entries corresponding to the measurement of v_y equal to 0, that is we define

$$K_0 = K_{LQR} \odot S\,,$$

   and try to use K_0 as our distributed controller.

b) Check that K_0 is stabilizing. What is the cost attained by K_0? Verify that the cost is higher than the LQR cost, as expected.

10. Apply Projected Gradient Descent, using K_0 as an initial controller.

    a) What cost can you achieve upon convergence to a stationary point $K^\star$?

    b) Plot the evolution of $x_1(t) = v_y(t)$ for the controlled system using K_{LQR}, K_0 and $K^\star$, for a chosen initial condition. Verify by inspection of the plots that the performance of $K^\star$ is superior to that of K_0 (see for instance Figure 2).

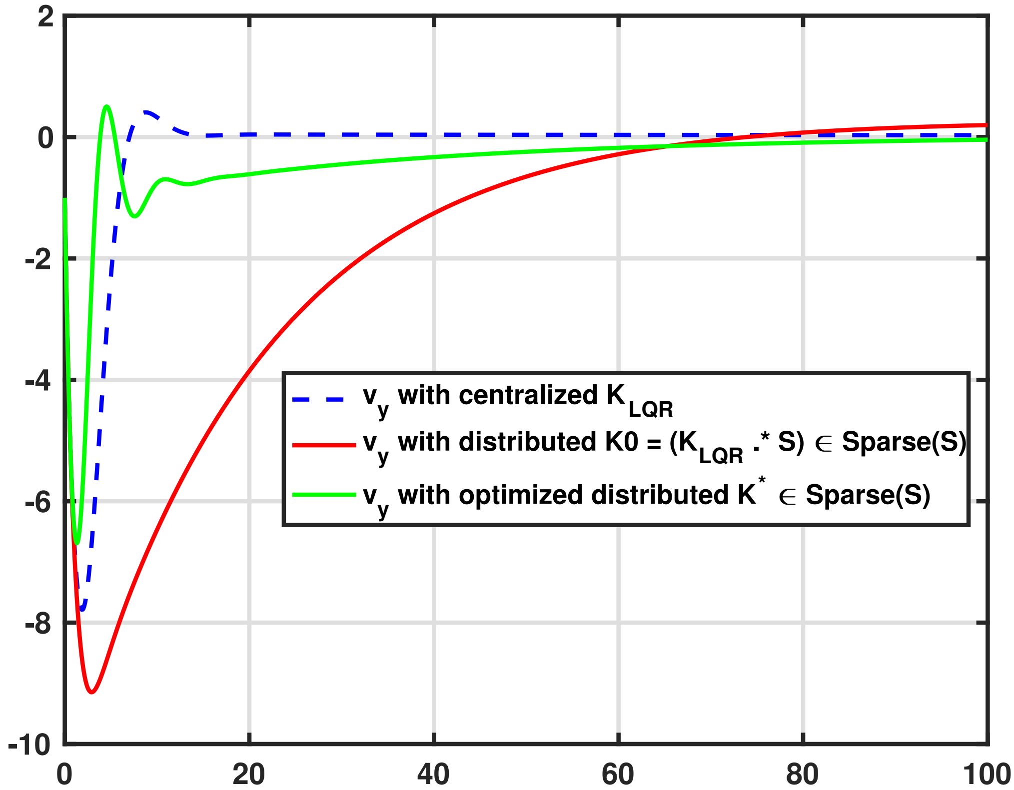

Figure 2: Controlled system using K_{LQR}, K_0 and $K^\star$