Multivariable Control (ME-422) - Exercise session 11B

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In the previous exercise session, we introduced the CSTR system...

In this set of exercises, you will learn to control a continuous stirred tank reactor (CSTR) around a given operating point.

CSTRs are very common in industry since they provide the adequate environment to make chemical reactions happen under precise system conditions (temperature, liquid concentration, etc.). Because of this, they need to be controlled to maintain the internal variables at the proper references despite the external disturbances. Precisely, the working behaviour of the considered system can be summarized as follows:

- The reactor receives an input liquid with a concentration c_f, a temperature T_f and a flow q. The concentration c_f is a variable that can be manipulated as a *control input* to the system. The temperature T_f and the flow q are not controlled. We first consider them as fixed and known parameters, but in a more general framework they could be considered as *external disturbances*, as we will do later in the course.
- The reactor walls are regulated to a temperature T_c which can be manipulated as a *control input* to the system.
- The internal reaction produces a fluid with a temperature T and the concentration c. The outflow rate is denoted by q.
- Both the concentration c and the temperature T in the reactor are measured variables.

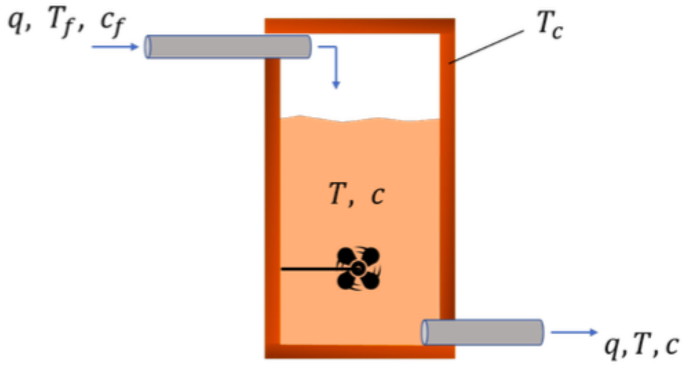

Figure 1: Schematic and realistic views of an industrial continuous stirred tank reactor.

Nonlinear system model The continuous-time nonlinear dynamics of the CSTR are described by the state-space equations:

$$\begin{aligned}
\dot{c}(t) &= \frac{q}{V}\,(c_f(t) - c(t)) - k_0\, e^{-\frac{E}{RT(t)}}\, c(t)\,, \\
\dot{T}(t) &= \frac{UA}{V\rho c_p}\,(T_c(t) - T(t)) + \frac{q}{V}\,(T_f - T(t)) - \frac{\Delta H}{\rho c_p} k_0\, e^{-\frac{E}{RT(t)}}\, c(t)\,.
\end{aligned} \tag{1}$$

Parameter	Value	Description (see Figure 1)	Units
V	100	Reactor volume	L
k_0	7.2×10^{10}	Nonthermal factor	$\frac{1}{\mathrm{min}}$
E	72747.5	Activation energy per mole	$\frac{\mathrm{J}}{\mathrm{mol}}$
R	8.314	Boltzmann's ideal gas constant	$\frac{\mathrm{J}}{\mathrm{mol\,K}}$
ΔH	5×10^4	Molar enthalpy (i.e. heat of reaction per mole)	$\frac{\mathrm{J}}{\mathrm{mol}}$
ρ	1000	Liquid density	g/L
c_p	0.239	Specific heat capacity of the liquid	$\frac{\mathrm{J}}{\mathrm{g\,K}}$
UA	5×10^4	Overall heat transfer coefficient multiplied by tank area	$\frac{\mathrm{J}}{\mathrm{min\,K}}$
q	100	Flow rate	$\frac{\mathrm{L}}{\mathrm{min}}$
T_f	350	Input temperature	K

Table 1: Parameters of the CSTR system.

Table 1 summarizes the physical meaning and values of the different parameters.

... and you were asked to:

1. Load the parameters from the `data_CSTR.m` file in the workspace. Then, implement the model given by (1) in Simulink.

2. Find the inputs $(\bar{c}_f, \bar{T}_c)$ that give the desired equilibrium at $\bar{c} = 0.5\,\frac{\mathrm{mol}}{\mathrm{L}}$, $\bar{T} = 350\,\mathrm{K}$. Corroborate the results by simulating the system at the equilibrium.

3. Simulate the system when adding a 1% disturbance over the initial conditions around the equilibrium. Does the system converge to previously computed equilibrium?

4. Linearize the system around $(\bar{c}, \bar{T})$ with inputs $(\bar{c}_f, \bar{T}_c)$, i.e. the equilibrium obtained in point 2. Is the linearized system stable? Can you conclude anything about the stability of the equilibrium of the nonlinear system?
   Hint: Use the time-based linearization block, for obtaining the matrices of the linearized system.

5. Multiple equilibria exist when considering the constant inputs $(\bar{c}_f, \bar{T}_c)$:

   (a) Load the data from the `CSTR_eq2.m` and verify that the state pair (`ctilde`, `Ttilde`) is an equilibrium when considering the inputs $(\bar{c}_f, \bar{T}_c)$.

   (b) Linearize the system around the new equilibrium $(\tilde{c}, \tilde{T}) = (\texttt{ctilde}, \texttt{Ttilde})$. Is the linearized system stable? Can you conclude anything about the stability of this new equilibrium of the nonlinear system? Validate your answer by simulating the system when adding a 1% perturbation on the initial conditions.

6. Discretize the linearized system around $(\bar{c}, \bar{T})$ using exact discretization method with sampling time $T_s = 0.05\,\mathrm{s}$. Simulate both the linearized continuous-time and the discrete-time models using Simulink. Plot the state trajectories of both systems and verify that they coincide at the sampling instants.
   Hint: use the Simulink blocks (discrete) state space and zero-order hold.

From now on, consider the discrete time system obtained in point 6 using the exact discretization method and $T_s = 0.05\,\mathrm{s}$.

7. Using full state information:

(a) Design a controller in order to assign both closed-loop eigenvalues at 0.85.

(b) In Simulink, close the loop to control the CSTR. Construct two different closed-loop systems using both the linearized and the nonlinear system models.

(c) Add a perturbation of +1% magnitude to the initial conditions of the systems. Simulate the evolution of the two closed-loop systems you just constructed, and compare the state trajectories of the two systems. Repeat the simulation with a perturbation of +4% magnitude. What can you observe?

(d) The *region of attraction* (ROA) of a nonlinear system is a safe subset of the state space in which a given controller renders an equilibrium point asymptotically stable. By sampling over different initial conditions, estimate a portion of the ROA inside the set

$$\left\{(c, T) : |c - \bar{c}| < 0.25\,\frac{\mathrm{mol}}{\mathrm{L}} \quad \text{and} \quad |T - \bar{T}| < 25\,\mathrm{K}\right\}.$$

8. Assume now that only measurements of the temperature are available. Is it possible to construct an observer to estimate the concentration? If yes, design a Luenberger observer with eigenvalues at $[0.5, 0.55]$ and test it in closed-loop with the controller designed in point 7.

In this exercise session, you are asked to:

9. Consider the nonlinear model of the CSTR as per (1), and assume now that q and T_f are no longer constant parameters but disturbances.

   (a) Obtain the new linearized and discretized model of the nonlinear system around the states $(\bar{c}, \bar{T})$ and inputs $(\bar{c}_f, \bar{T}_c, \bar{q}, \bar{T}_f)$, when considering q and T_f as disturbance inputs. Use the exact discretization method and $T_s = 0.05\,\mathrm{s}$.

   (b) Assume an unknown but constant disturbances act on the plant states through q and T_f. Design a controller with integral action to achieve asymptotic tracking of constant references. For the controller design, solve an eigenvalue assignment problem using pole placement with poles at `[0.85, 0.85, 0.5, 0.5]`.

   (c) Implement and validate the designed controller architecture in Simulink using both the linearized model and the nonlinear one. Track constant references signal of $\pm 2\%$ of the operation states $(\bar{c}, \bar{T})$ and consider perturbations of $\pm 5\%$ around the operation values $(\bar{q}, \bar{T}_f)$.

10. Replace the controller designed in the previous point by pole placement with an LQR design. Start by setting

$$Q = \mathrm{diag}\left(\frac{1}{\bar{c}}, \frac{1}{\bar{T}}, \frac{1}{\bar{c}}, \frac{1}{\bar{T}}\right) \qquad \text{and} \qquad R = \mathrm{diag}\left(\frac{1}{\bar{c}_f}, \frac{1}{\bar{T}_c}\right).$$

   It can be seen that the *settling-time* is very large. By changing a single entry in Q or R, design a new controller for improving tracking.