

Exercises of Chapter 8

Digital Control

Problem 8.1 The impulse response $\{g(kh)\}$ of an LTI system is given in Figure 1. Graphically determine using convolution the response of the system when excited by the signal $\{1, 1, 1, 1, 0, 0, \dots\}$.

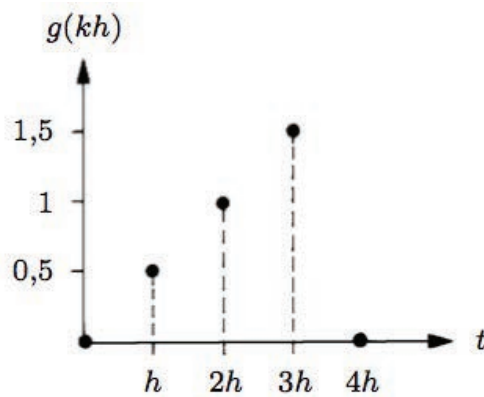


Figure 1 - Impulse response of an LTI system

Problem 8.2 A controller should be designed for a system to achieve a closed-loop bandwidth of 10 rad/s. Propose a sampling period and a forth-order butterworth anti-aliasing filter. Compute the sampling frequency and a forth-order butterworth anti-aliasing filter if the rise-time of the system is 20 ms.

Remark: Note that the anti-aliasing filter should pass all signals up to the Nyquist frequency (half of the sampling frequency) and cut every higher frequency signals. Therefore the ideal bandwidth for the anti-aliasing filter is the Nyquist frequency. However, as the ideal filter is not realizable with a finite order transfer function, the cut-off frequency of the anti-aliasing filter is chosen smaller than the Nyquist frequency (to be sure that higher frequencies are well filtered) and larger than the bandwidth of the system (to be sure that the signals in the bandwidth are not attenuated). A butterworth filter can be designed in Matlab using `butter`. A forth-order

butterworth filter with a cutoff frequency of ω_c rad/s is given by:

$$F(s) = \frac{1}{\left(\frac{s}{\omega_c}\right)^4 + 2.613 \left(\frac{s}{\omega_c}\right)^3 + 3.411 \left(\frac{s}{\omega_c}\right)^2 + 2.613 \left(\frac{s}{\omega_c}\right) + 1}$$

Problem 8.3 Z-Transform Exercises

(a) Compute the Z transform of the signal shown in Figure 2.

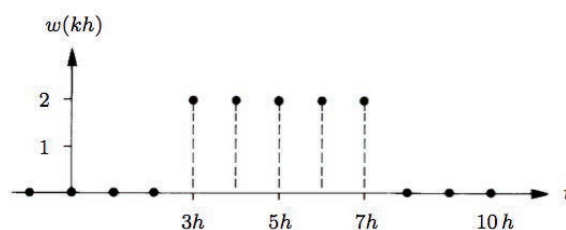


Figure 2 - Discrete-time signal

(b) Compute the Z transform of the signal defined by $w(kh) = \frac{1}{2}(kh)^2, k \geq 0$.

(c) Find using a partial fraction expansion the inverse Z transform of

$$W(z) = \frac{z^2 - 3z + 8}{(z - 2)(z + 2)(z + 3)}$$

(d) Find using numerical inversion the inverse Z transform of

$$W(z) = \frac{z + 3}{z^2 - 3z + 2}$$

The calculation of samples $w(kh), k = 0, 1, \dots, 4$ should be done using the theorem given on Chapter 8, Slide 68.

(e) Apply the final value theorem to the signal $e^{-akh} \sin(\omega kh), k \geq 0$ where a is a real number strictly larger than zero. What happens when $a \leq 0$?

Problem 8.4 The discrete-time transfer function for a system is

$$G(z) = \frac{0.393}{z - 0.607}$$

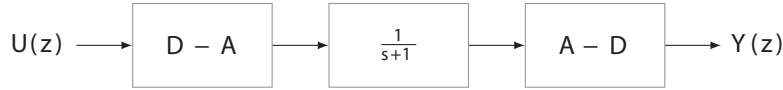
Determine its step response.

Problem 8.5 The position of a brushless motor is controlled via a current, and the behaviour of the system is approximately like a double integrator :

$$G(s) = \frac{\gamma}{s^2} \quad \gamma > 0 \quad (1)$$

Determine the discrete-time transfer function of this process using ZOH method for a sampling time h .

Problem 8.6 Consider the following block diagram:



Compute the transfer function $Y(z)/U(z)$. Deduce its discrete time step response or the response $y(kh)$ when $u(kh) = 1, k \geq 0$. Determine the step response for the isolated analog element, which has the transfer function $\frac{1}{s+1}$, then sample this signal. Why is this result equal to the step response of the system given in the diagram above? Is this true for any signal $u(kh)$ or any analog element placed between the converters?

Problem 8.7 Compute discrete approximations of the following controllers using the Euler (backward), Tustin and zero-pole matching methods:

$$K(s) = \frac{1}{s}$$

$$K(s) = \frac{s}{s^2 + 2}$$

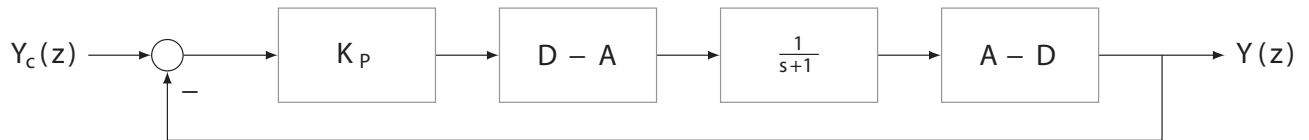
$$K(s) = \frac{s + 4}{s + 3}$$

$$K(s) = \frac{1}{s(s + 8)}$$

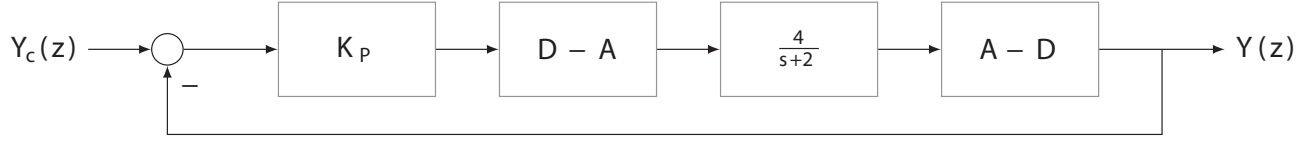
Problem 8.8 Compute a continuous approximation of the following system sampled at $h = 1s$ using the Euler (backward) and Tustin methods:

$$G(z) = \frac{0.04762z + 0.04762}{z - 0.9048}$$

Problem 8.9 For the process in the figure below, determine the values of the sampling period h for which this closed-loop system is BIBO stable.



Problem 8.10 The system given below is a motor whose speed is regulated by a proportional controller with gain $K_p > 0$.



If the sampling period is $h = 0.025$ s, compute the ultimate gain of the discrete-time model.

Problem 8.11 Consider a PI controller in a unity feedback system as:

$$K(s) = K_P + \frac{K_I}{s}$$

Find a discrete representation of the PI controller $K(q^{-1})$ and an equivalent RST digital controller.

Problem 8.12 Consider the following plant model:

$$G(z) = \frac{0.5(z - 0.8)}{(z - 0.7)(z - 0.5)}$$

Take $P_d(q^{-1}) = 1 - 0.7q^{-1}$ and the same dynamics for tracking and regulation.

1. Compute an RST controller.
2. Redesign the RST controller such that it includes an integrator.

Problem 8.13: Consider the following plant model:

$$G(q^{-1}) = \frac{0.5q^{-1} - 0.4q^{-2}}{1 - 1.2q^{-1} + 0.35q^{-2}}$$

For the regulation dynamic choose the dominant closed loop pole to have a damping of $\zeta = 0.8$ and a natural frequency of $\omega_n = 1$ rad/s. Note that $h = 1$ s.

1. Design an RST controller for Model Reference Control problem.
2. Add a fixed term $H_R(q^{-1}) = 1 + q^{-1}$ in the controller.

Problem 8.14: Consider the following plant model:

$$G(q^{-1}) = \frac{0.5q^{-1} - 0.4q^{-2}}{1 - 1.2q^{-1} + 0.35q^{-2}}$$

Design an RST controller that includes an integrator using the internal model control (IMC).

Problem 8.15: Consider the following plant model:

$$G(s) = \frac{2.6e^{-0.25s}}{s + 1.3}$$

1. Compute a discret-time model for the system using the ZOH method ($h = 0.125$)
2. Compute an RST controller with integrator to place the closed-loop pole of the system at 0.8 (consider the same dynamic for tracking and regulation).

Problem 8.16: Consider the following system

$$H(z) = \frac{4(0.8 - z)}{z(0.5 - z)(1.8 - z)}$$

with $h = 0.1$ s.

- a) Compute an RST controller and a reference model using the MRC approach.
- The reference model is a first order model with unity steady state gain and a bandwidth of 2 rad/s (use ZOH method).
 - The regulation dynamic is defined by a natural frequency of $\omega_n = 4$ and a damping factor of $\zeta = 0.8$.
 - The controller includes an integrator.