Exercises of Chapter 7
State-Space Methods

7. Show that the transfer function is not changed by a linear transformation of state.

15. Given the system,

$$\dot{\mathbf{x}} = \begin{bmatrix} -5 & 1 \\ -2 & -1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u,$$

with zero initial conditions, find the steady-state value of $\mathbf{x}$ for a step input u.

16. Consider the system shown in Fig. 7.85:

a) Find the transfer function from U to Y.

b) Write state equations for the system using the state variables indicated.

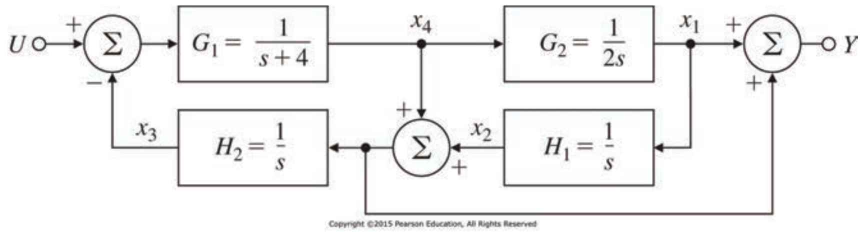

Figure 7.85 : Block diagram of the system in problem 7.16

22. For the system,

$$\begin{aligned} \dot{\mathbf{x}} &= \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}, \end{aligned}$$

design a state feedback controller that satisfies the following specifications:

- Closed-loop poles have a damping coefficient $\zeta = 0.707$.
- Step-response peak time is under 3.14 sec.

25. Consider the system in Fig. 7.87.
a) Write a set of equations that describes this system in the control canonical form as $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$ and $y = \mathbf{C}\mathbf{x}$.
b) Design a control law of the form,

$$u = -\begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

which will place the closed-loop poles at $s = -2 \pm 2j$.

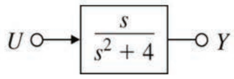

Figure 7.87: System for Problem 7.25.

34. Consider the system

$$\mathbf{A} = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{C} = [1 \;\; 2],$$

and assume that you are using feedback of the form $u = -\mathbf{K}\mathbf{x} + r$, where r is a reference input signal.
a) Show that $(\mathbf{A},\mathbf{C})$ is observable.
b) Show that there exists a $\mathbf{K}$ such that $(\mathbf{A} - \mathbf{BK},\mathbf{C})$ is unobservable.

c) Compute a $\mathbf{K}$ of the form $\mathbf{K} = [1, K_2]$ that will make the system unobservable as in part (b); that is, find K_2 so that the closed-loop system is not observable.
d) Compare the open-loop transfer function with the transfer function of the closed-loop system of part (c). What is the unobservability due to?

37. Consider the electric circuit shown in Fig. 7.90.
a) Write the internal (state) equations for the circuit. The input $u(t)$ is a current, and the output y is a voltage. Let $x_1 = i_L$ and $x_2 = v_c$.
b) What condition(s) on R, L, and C will guarantee that the system is controllable?
c) What condition(s) on R, L, and C will guarantee that the system is observable?

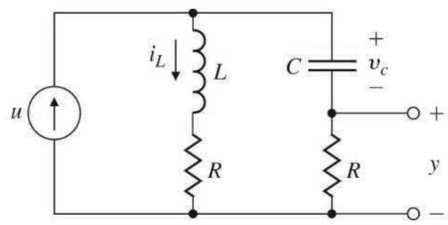

Figure 7.90: Electric circuit for Problem 7.37.

46. The linearized equations of motion of the simple pendulum in Fig. 7.96 are

$$\ddot{\theta} + \omega^2 \theta = u.$$

a) Write the equations of motion in state-space form.
b) Design an estimator (observer) that reconstructs the state of the pendulum given measurements of $\dot{\theta}$. Assume $\omega = 5$ rad/sec, and pick the estimator roots to be at $s = -10 \pm 10j$.
c) Write the transfer function of the estimator between the measured value of $\dot{\theta}$ and the estimated value of θ.
d) Design a controller (that is, determine the state feedback gain $\mathbf{K}$) so that the roots of the closed-loop characteristic equation are at $s = -4 \pm 4j$.

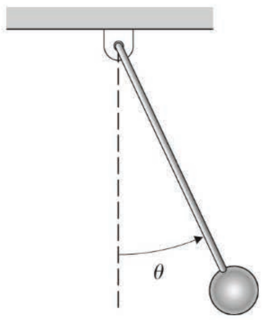

Figure 7.96: Pendulum diagram for Problem 7.46.

48. A certain process has the transfer function $G(s) = 4/(s^2 - 4)$.
a) Find $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ for this system in observer canonical form.
b) If $u = -\mathbf{Kx}$, compute $\mathbf{K}$ so that the closed-loop control poles are located at $s = -2 \pm 2j$.
c) Compute $\mathbf{L}$ so that the estimator-error poles are located at $s = -10 \pm 10j$.
d) Give the transfer function of the resulting controller (for example, using Eq. (7.177)).
e) What are the gain and phase margins of the controller and the given open-loop system?

51. Consider the control of

$$G(s) = \frac{Y(s)}{U(s)} = \frac{10}{s(s+1)}.$$

a) Let $y = x_1$ and $\dot{x}_1 = x_2$, and write state equations for the system.
b) Find K_1 and K_2 so that $u = -K_1 x_1 - K_2 x_2$ yields closed-loop poles with a natural frequency $\omega_n = 3$ and a damping ratio $\zeta = 0.5$.
c) Design a state estimator for the system that yields estimator error poles with $\omega_{n1} = 15$ and $\zeta_1 = 0.5$.
d) What is the transfer function of the controller obtained by combining parts (a) through (c)?

58. Consider a system with state matrices,

$$\mathbf{A} = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \ \mathbf{C} = [\ 1 \quad 3\ ].$$

a) Use feedback of the form $u(t) = -\mathbf{Kx}(t) + \bar{N}r(t)$, where $\bar{N}$ is a nonzero scalar, to move the poles to $-3 \pm 3j$.
b) Choose $\bar{N}$ so that if r is a constant, the system has zero steady-state error; that is $y(\infty) = r$.
c) Show that if $\mathbf{A}$ changes to $\mathbf{A} + \delta\mathbf{A}$, where $\delta\mathbf{A}$ is an arbitrary 2×2 matrix, then your choice of $\bar{N}$ in part (b) will no longer make $y(\infty) = r$. Therefore, the system is not robust under changes to the system parameters in $\mathbf{A}$.
d) The system steady-state error performance can be made robust by augmenting the system with an integrator and using unity feedback; that is, by setting $\dot{x}_I = r - y$, where x_I is the state of the integrator. To see this, first use state feedback of the form $u = -\mathbf{Kx} - K_1 x_I$ so that the poles of the augmented system are at $-3, -2 \pm j\sqrt{3}$.
e) Show that the resulting system will yield $y(\infty) = r$ no matter how the matrices $\mathbf{A}$ and $\mathbf{B}$ are changed, as long as the closed-loop system remains stable.

60. Consider the following system :

$$G(s) = \frac{s + \alpha}{s^2}$$

1. Give a state-space model of the system in control canonical form. Is this representation observable for all α ?
2. Compute a state feedback controller, $\boldsymbol{K}$, using the LQR method with $Q = I$ and $R = 1$.
3. Compute a state observer, $\boldsymbol{L}$, with $\alpha = 1$ to place all observer poles at -3.

61. Given the following system :

$$G(s) = \frac{s + 10}{s^2}$$

a) Compute a state feedback controller with integral action, $\boldsymbol{K}$, such that the closed-loop poles are all at -2. This controller gives a zero tracking error $e(t) = r(t) - y(t)$ for a step signal $r(t)$.

b) Assuming that all states are measured, determine a state space representation for the closed-loop system :
- between the output disturbance $w(t)$ and the output $y(t) = \boldsymbol{Cx}(t) + w(t)$.
- between $r(t)$ and $u(t)$.