Exercises of Chapter 6
Frequency Response Methods

6.1 The magnitude Bode plot of a transfer function

$$G(s) = \frac{K(1+0.5s)(1+as)}{s(s/8+1)(bs+1)(s/36+1)}$$

is given in the following figure. Determine K, a and b from the plot.

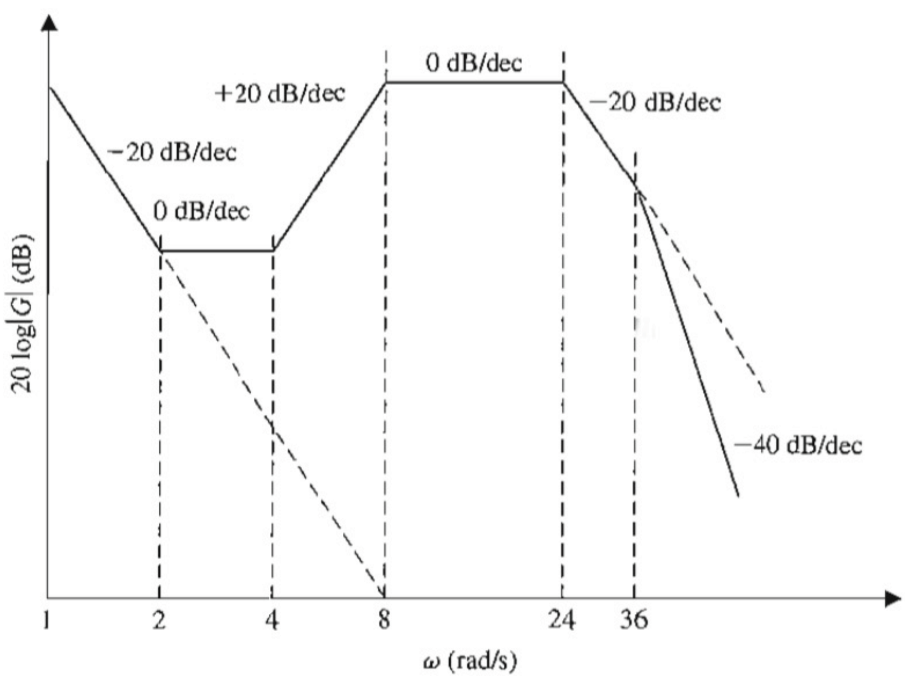

Fig. 6.1: The Bode diagram of $G(s)$

6.2 The Bode diagram of a system is given below. Find an approximate model for the system.

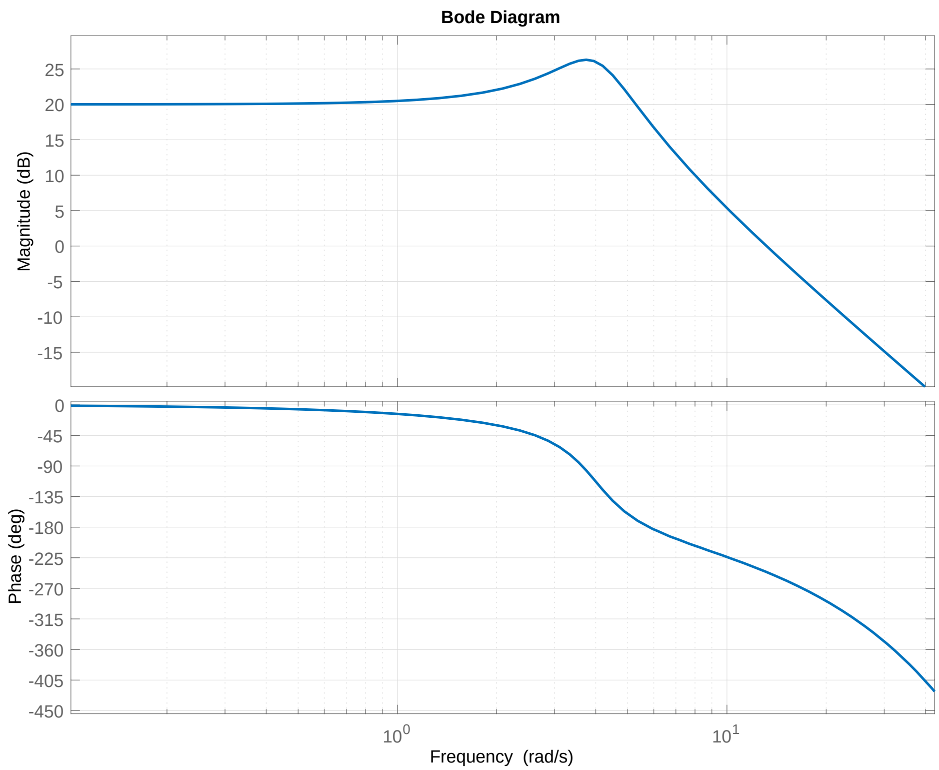

Fig. 6.2: The Bode diagram of $G(s)$

6.3 Suppose that

$$L(s) = \frac{k}{s(0.1s+1)(s+1)}$$

Sketch the Bode diagram of $L(s)$ for $k = 1$.

(a) Find the value of k to give a gain margin of 10dB.

(b) Find the value of k to give a gain margin of 30dB.

(c) Find the value of k to give a phase margin of 60°.

6.4 The Bode diagram of $G(s) = \dfrac{\gamma}{s(s+\alpha)}$ is given in Fig. 6.4.

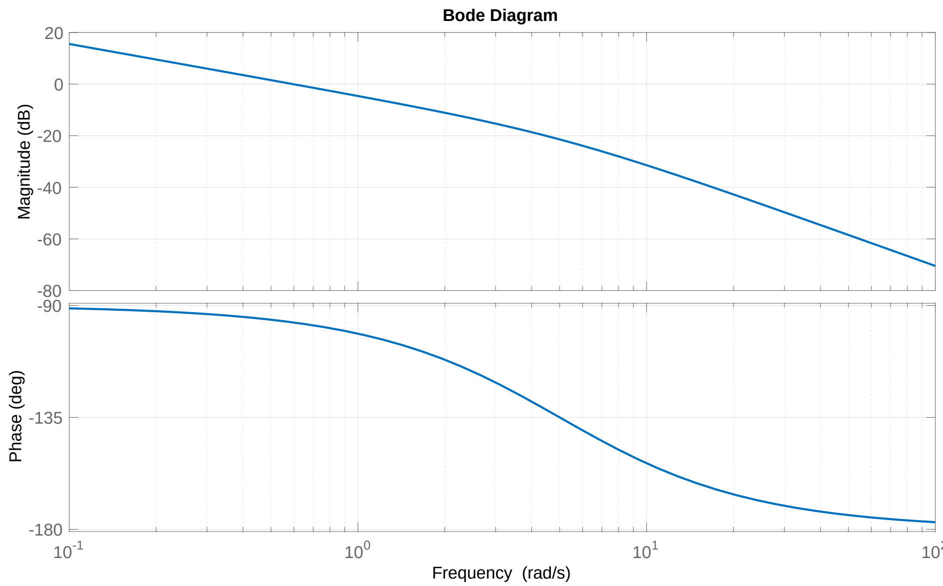

Fig. 6.4: The Bode diagram of $G(s)$

1. Determine α and γ.
2. Compute a controller using the model reference approach to obtain an overshoot of about 30% for a step reference and a closed-loop bandwidth of 2 rad/s.
3. Compute the steady-state value of the output for a unit step disturbance at the input of the plant.

6.5 The open-loop transfer function of a unity feedback system is:

$$L(s) = \frac{5}{s(1+0.5s)(1+0.1s)}$$

The Bode plot of the open-loop system is given as:

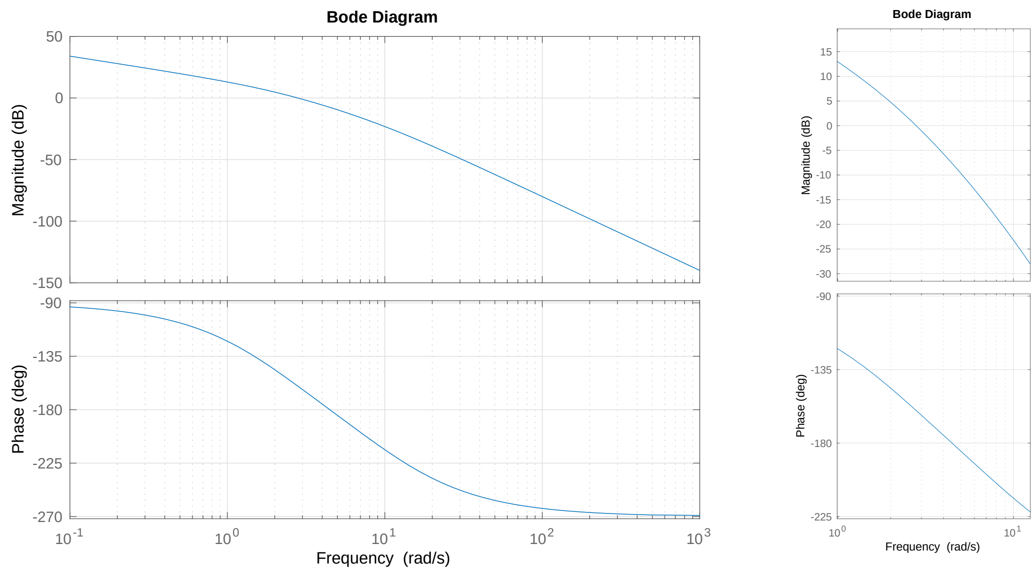

Figure 6.5: The Bode diagram of the open-loop transfer function (left), with a zoom on the interesting part of the Bode diagram (right)

(a) Find the crossover frequency, phase margin and gain margin.

(b) If the steady state gain is doubled, find crossover frequency, gain margin and phase margin.

6.6 The transfer function of the plant model and the feedback controller are given as:

$$G(s) = \frac{e^{-0.1s}}{s+10} \qquad D_c(s) = K_p$$

Select the proportional gain K_p so that the phase margin of the system is 50°. Determine the gain margin for the selected K_p.

6.7 The following system is regulated by a proportional controller with gain K_p.

$$G(s) = \frac{4s+1}{s(s-1)}$$

If $K_p = 1$, draw the Nyquist plot (feel free to use Matlab). Demonstrate the closed loop BIBO stability with the help of the Nyquist criterion. Then, calculate the set of gains K_p for which the closed-loop system is stable.

6.8 The Nyquist plot for an open-loop stable system resembles the one shown in Fig.6.8. What are the gain and phase margin(s) for the system given that $\alpha = 0.4$, $\beta = 1.3$ and $\phi = 40°$. Describe what happens to the stability of the system as the gain goes from zero to a very large value. Also sketch what the corresponding Bode plots would look like for the system.

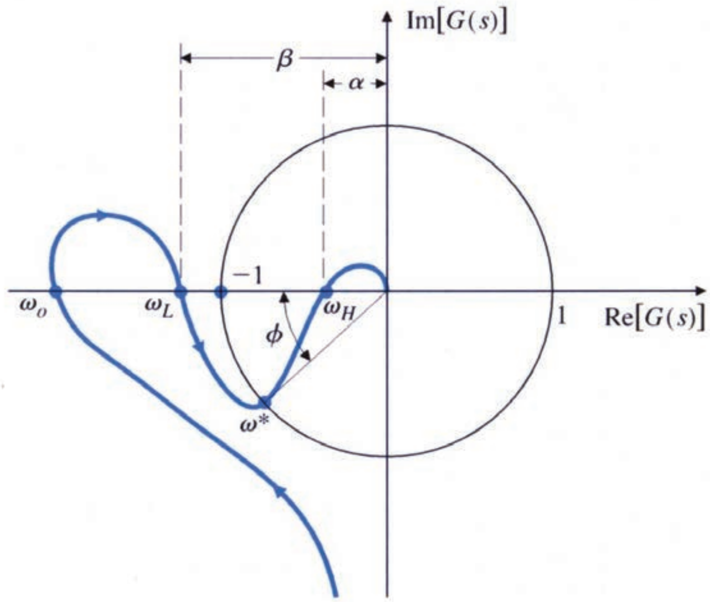

Fig. 6.8: The Nyquist diagram of $G(s)$

6.9 For a given system, show that how the ultimate period P_u and the corresponding ultimate gain K_u for the Ziegler-Nichols method can be found using the following: (a) Nyquist diagram (b) Bode plot (c) root locus.

6.10 The unit step response of a system is depicted in the following figure:

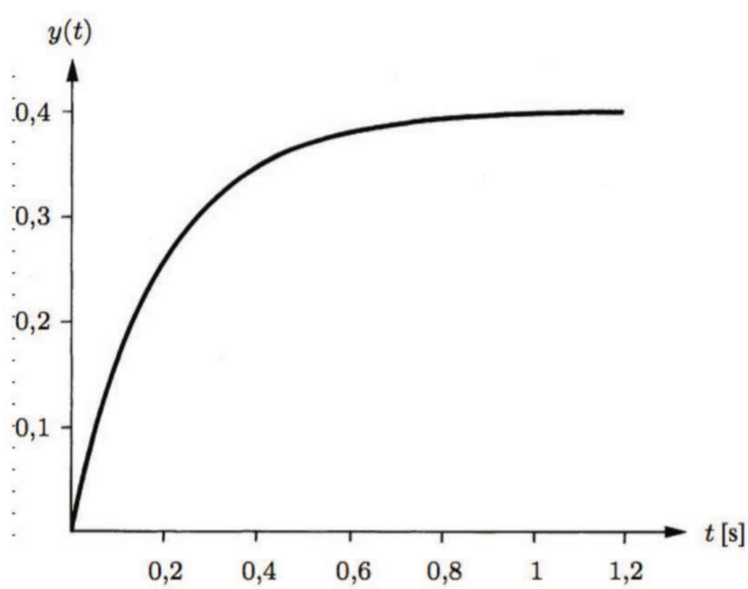

Figure 6.10 - Unit step response

The transfer function writes:

$$G(s) = \frac{b}{s+a}$$

Based on Fig. 6.10, estimate the constants a and b and sketch its Bode plot. Design a regulator such that the steady-state error (for a step reference) is zero, the phase margin larger or equal to 60° and the crossover frequency equal to 10 rad/s.

6.11 The transfer function of a system is given as:

$$G(s) = 0.043\frac{(s+11.43)(2-s)}{s(s+0.86)}$$

Sketch the Bode diagram (magnitude and phase) of the system. Design a regulator such that the steady-state error for tracking a reference step signal is zero, the phase margin larger than or equal to 60° and the crossover frequency is equal to 0.3 rad/s.

6.12 The magnitude Bode diagram of $G(s)$ is represented in Fig. 6.12:

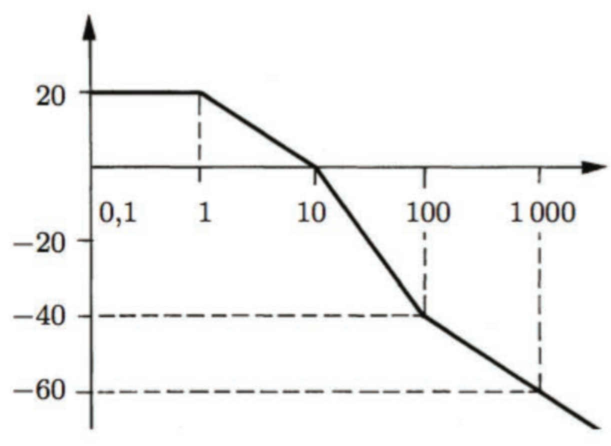

Figure 6.12 - Magnitude Bode diagram

A closed-loop bandwidth of at least $\omega_{BW} = 30$ rad/s and no steady-state error (for a step reference) is desired. Design a controller $D_c(s)$ to achieve the desired performance. Repeat the problem with $\omega_{BW} = 300$ rad/s.

6.13 Given a speed-controlled electric motor described by the following transfer function

$$G(s) = \frac{\gamma}{\tau s + 1} \qquad \gamma = 0.89, \quad \tau = 0.28$$

Synthesize a controller imposing the following specifications: zero steady-state error for step disturbance, phase margin larger than or equal to 60° and a bandwidth of at least 12 rad/s for the closed-loop system.

6.14 The plant transfer function is given by:

$$G(s) = \frac{40}{s(s+2)}$$

Compute a controller for

- a steady-state error to a ramp reference signal of less than 0.05,
- a phase margin of 45°,
- a closed-loop bandwidth of at least 10 rad/s.

6.15 Given the Bode diagram of $G(s)$ in the following figure:

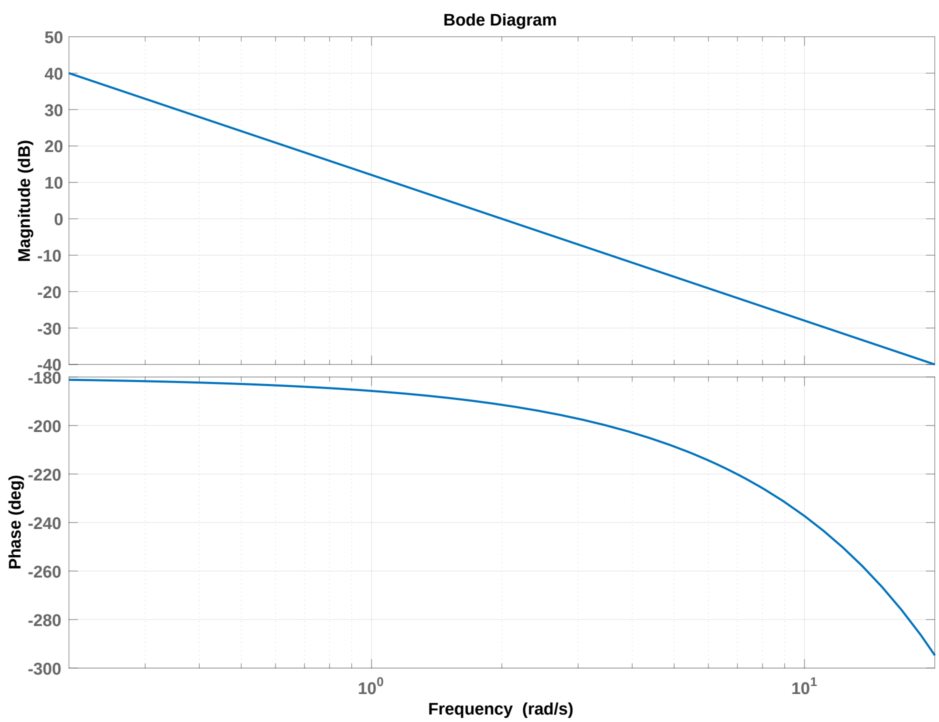

Fig. 6.15: The Bode diagram of $G(s)$

1. Determine the transfer function $G(s)$.
2. Compute a lead compensator:

$$D_c(s) = \frac{1+\tau\alpha s}{1+\tau s}$$

To guarantee a phase margin of 45° and a closed-loop band width of at least 3.5 rad/s.