Exercises of Chapter 4
Feedback Control Systems

4. A unity feedback control system has the open-loop transfer function

$$G(s) = \frac{A}{s(s+a)}.$$

   (a) Compute the sensitivity of the closed-loop transfer function to changes in the parameter A.

   (b) Compute the sensitivity of the closed-loop transfer function to changes in the parameter a.

   (c) If the unity gain in the feedback changes to a value of $\beta \neq 1$, compute the sensitivity of the closed-loop transfer function with respect to β.

10. Consider the system shown in Fig. 4.28, where

$$D_c(s) = K\frac{(s+\alpha)^2}{s^2+\omega_o^2}.$$

   (a) Prove that if the system is stable, it is capable of tracking a sinusoidal reference input $r = \sin\omega_o t$ with zero steady-state error. (Look at the transfer function from R to E and consider the gain at ω_o.)

   (b) Use Routh's criterion to find the range of K such that the closed-loop system remains stable if $\omega_o = 1$ and $\alpha = 0.25$.

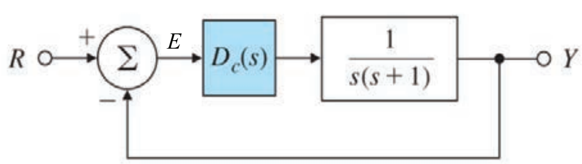


Figure 4.28: Control system for Problem 4.10

15. A controller for a satellite attitude control with transfer function $G(s) = 1/s^2$ has been designed with a unity feedback structure and has the transfer function:

$$D_c(s) = \frac{10(s+2)}{s+5}$$

(a) Find the steady state error for tracking unit step, unit ramp and unit parabolic signals.

(b) If a disturbance torque adds to the control so that the input to the process is $u + w$, what is the steady-state error for a step disturbance.

16. A compensated motor position control system is shown in Fig. 4.31. Assume that the sensor dynamics are $H(s) = 1$.

(a) Can the system track a step reference input r with zero steady-state error? Compute the steady-state error for tracking a unit ramp reference input.

(b) Can the system reject a step disturbance w with zero steady-state error?

(c) Compute the sensitivity of the closed-loop transfer function to changes in the plant pole at -2.

(d) In some instances there are dynamics in the sensor. Repeat parts (a) to (b) for $H(s) = 20/(s+20)$ and compare the corresponding steady-state errors.

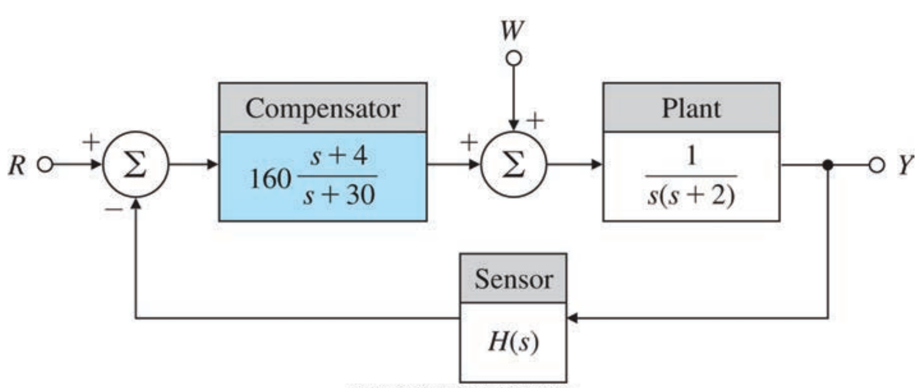
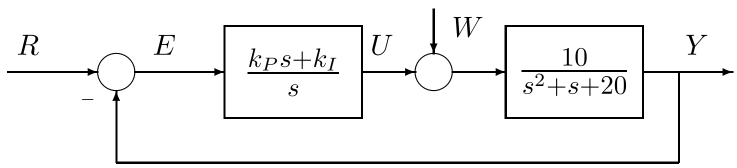


Fig. 4.31 Control system for Problem 4.16

30. Consider the following system with a PI controller:



(a) Determine the transfer function from R to Y.

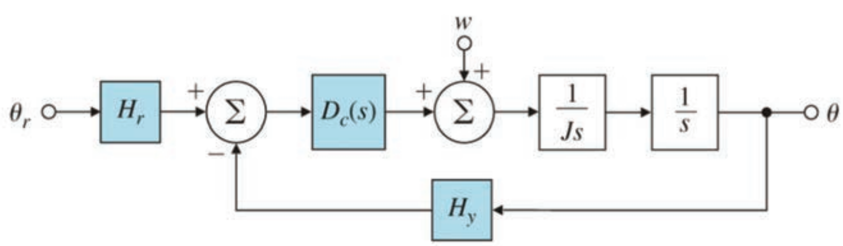
(b) Determine the transfer function from W to Y.

(c) Compute the steady-state error for tracking a step and a ramp reference signal.

(d) Compute the steady-state error for a step and a ramp disturbance signal.

(e) For which values of k_P and k_I, the closed-loop system is stable?

34. Consider the satellite-attitude control problem shown in the following figure:



where the normalized parameters are:

$J = 10$	spacecraft inertia, Nms^2/rad
θ_r	reference satellite attitude, rad
θ	actual satellite attitude, rad
$H_y = 1$	sensor scale factor, volt/rad
$H_r = 1$	reference sensor scale factor, volts/rad
w	disturbance torque, Nm

(a) Use proportional control $D_c(s) = k_P$, and give the range of values for k_P for which the system will be stable.

(b) Use PD control and let $D_c(s) = k_P + k_D s$ and determine the steady state error for a step reference signal and a step disturbance signal.

(c) Use PI control and let $D_c(s) = k_P + k_I/s$ and determine the steady state error for a step reference signal and a step disturbance signal.

(d) Use PID control and let $D_c(s) = k_P + k_I/s + k_D s$ and determine the steady state error for a step reference signal and a step disturbance signal.

36. The unit-step response of a paper machine is shown in Fig. 4.47(a) where the input into the system is stock flow onto the wire and the output is basis weight (thickness). The time delay and slope of the transient response may be determined from the figure.

   (a) Find the proportional, PI, and PID-controller parameters using the Zeigler–Nichols transient-response method.

   (b) Using proportional feedback control, control designers have obtained a closed-loop system with the unit impulse response shown in Fig. 4.47(b). When the gain $K_u = 8.556$, the system is on the verge of instability. Determine the proportional-, PI-, and PID-controller parameters according to the Zeigler–Nichols ultimate sensitivity method.

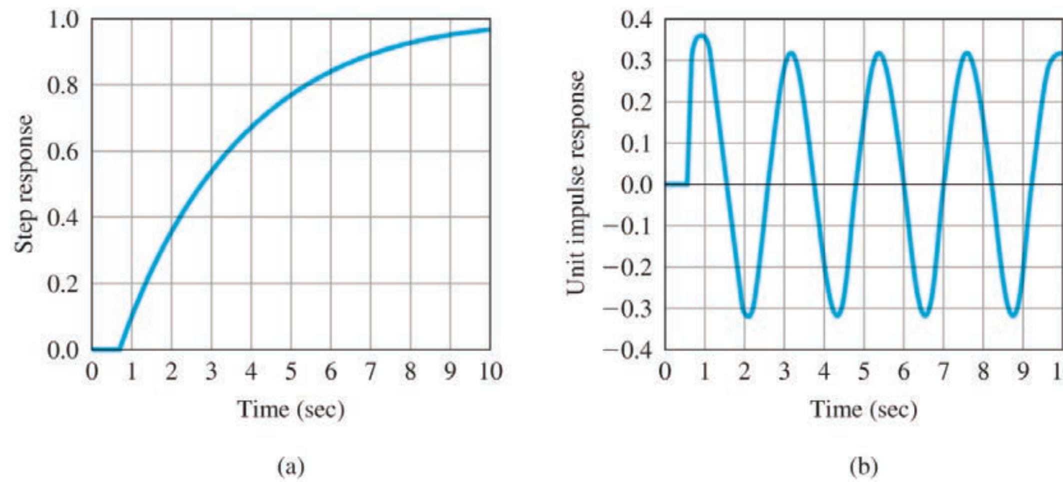


Figure 4.47: Paper-machine response data for Problem 4.36

37. A paper machine has the transfer function

$$G(s) = \frac{e^{-2s}}{3s+1},$$

where the input is stock flow onto the wire and the output is basis weight or thickness.

(a) Find the PID-controller parameters using the Zeigler–Nichols tuning rules.

(b) The system becomes marginally stable for a proportional gain of $K_u = 3.044$ as shown by the unit impulse response in Fig. 4.48. Find the optimal PID-controller parameters according to the Zeigler–Nichols tuning rules.

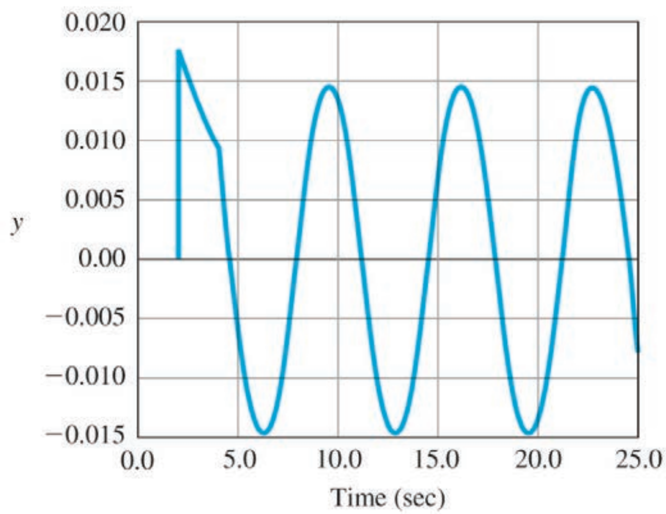


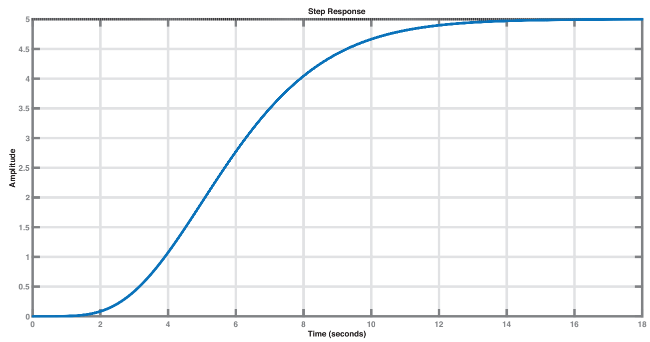
Figure 4.48: Unit impulse response for paper-machine in Problem 4.37

41. Consider the following dynamic system :

$$G(s) = \frac{e^{-30s}}{100s+1}$$

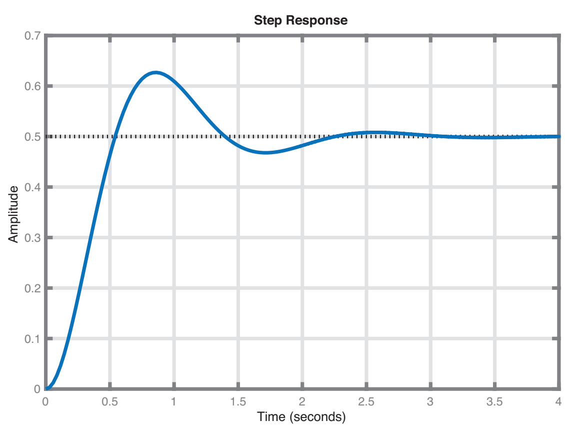
1. Compute the step response, and design by the first method of Ziegler-Nichols P, PI and PID controllers.
2. Which terms (P, I and D) should be included in the controller? Explain.
3. For the proportional controller, compute the steady-state error in response to a step reference signal.

42. The step response of a system is given.



1. Compute a PID controller using the ZN method.
2. Identify the parameters of a first-order model with delay
$$G(s) = \frac{\gamma e^{-\theta s}}{\tau s + 1}$$
such that its step response is close to that of the system.
3. Compute a PID controller using the model reference control method such that the closed-loop system has a settling time of 10 s.

43. The unit step response of a system is given and its transient parameters are measured:



RiseTime:	0.3663
SettlingTime:	2.1023
SettlingMin:	0.4533
SettlingMax:	0.6269
Overshoot:	25.3741
Undershoot:	0
Peak:	0.6269
PeakTime:	0.8635

1. Identify the parameters of a second-order model
$$G(s) = \frac{\gamma \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$
such that its step response is close to that of system.
2. Compute a PID controller using the model reference control method such that the closed-loop system has a bandwidth of $1.2\omega_n$.