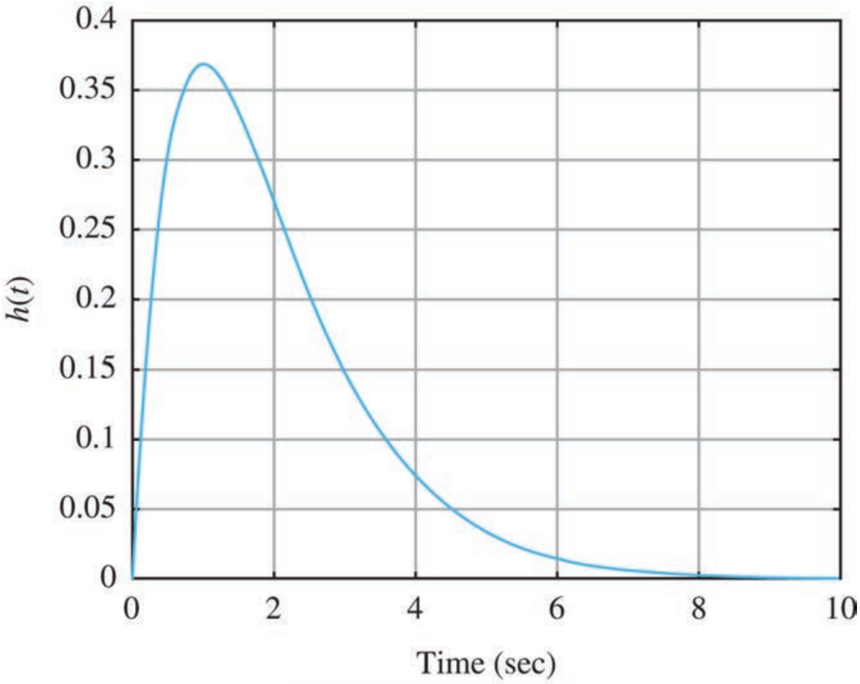
Exercices For Chapter 3
Analysis of Dynamic Systems

10. Using the convolution integral, find the step response of the system whose impulse response is given below and shown in Figure 3.44:

$$h(t) = \begin{cases} 2te^{-2t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$



17. For a second-order system with transfer function

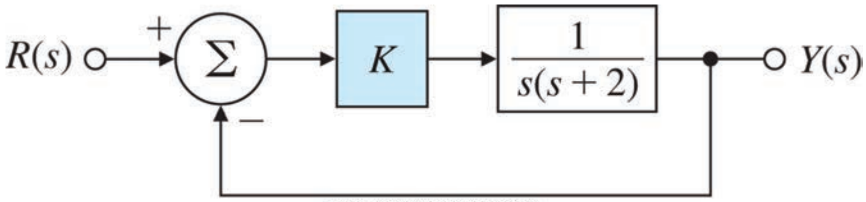
$$G(s) = \frac{2}{s^2 + s - 2},$$

determine the following:

(a) The DC gain;

(b) The final value to a unit step input.

26. For the unity feedback system shown in Fig. 3.54, specify the gain K of the proportional controller so that the output $y(t)$ has an overshoot of no more than 10% in response to a unit step.



27. For the unity feedback system shown in Fig. 3.55, specify the gain and pole location of the compensator so that the overall closed-loop response to a unit-step input has an overshoot of no more than 25%, and a 1% settling time of no more than 0.1 sec.

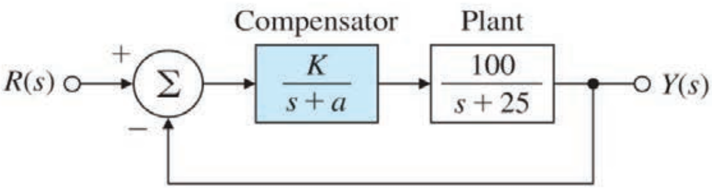
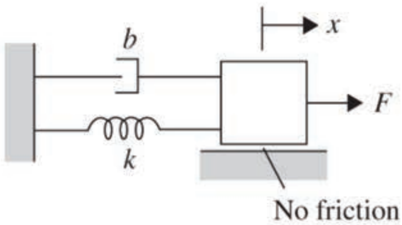


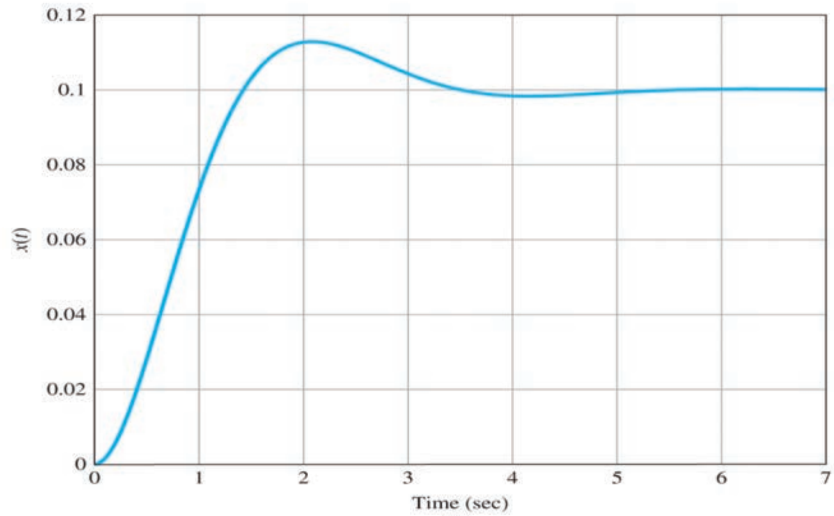
Figure 3.55: Unity feedback system for Problem 3.27

33. A simple mechanical system is shown in Fig. 3.58 (a). The parameters are k = spring constant, b = viscous friction constant, m = mass. A step of 2 Newtons force is applied as $f = 2 \times\ 1(t)$ and the resulting step response is shown in Fig. 3.58 (b). What are the values of the system parameters k, b, and m?



(a)

Figure 3.58: (a) Mechanical system for Problem 3.33



(b)

(b) Step response for Problem 3.33

35. The equations of motion for the DC motor shown in Fig. 2.32 were given in Eqs. (2.62-63) as

$$J_m\ddot{\theta}_m + \left(b + \frac{K_t K_e}{R_a}\right)\dot{\theta}_m = \frac{K_t}{R_a}v_a.$$

Assume that

$$\begin{aligned} J_m &= 10\ \mathrm{kg}\cdot\mathrm{m}^2, \\ b &= 1\ \mathrm{N}\cdot\mathrm{m}\cdot\mathrm{sec}, \\ K_e &= 2\ \mathrm{V}\cdot\mathrm{sec}, \\ K_t &= 2\ \mathrm{N}\cdot\mathrm{m/A}, \\ R_a &= 10\Omega\ . \end{aligned}$$

(a) Find the transfer function between the applied voltage v_a and the motor speed $\dot{\theta}_m$.

(b) What is the steady-state speed of the motor after a voltage $v_a = 10$ V has been applied?

(c) Find the transfer function between the applied voltage v_a and the shaft angle θ_m.

(d) Suppose feedback is added to the system in part (c) so that it becomes a position servo device such that the applied voltage is given by

$$v_a = K(\theta_r - \theta_m),$$

where K is the feedback gain. Find the transfer function between θ_r and θ_m.

(e) What is the maximum value of K that can be used if an overshoot $M_p < 20\%$ is desired?

(f) What values of K will provide a rise time of less than 4 sec? (Ignore the M_p constraint.)

36. You wish to control the elevation of the satellite-tracking antenna shown in Figs. 3.60 and 3.61. The antenna and drive parts have a moment of inertia J and a damping B; these arise to some extent from bearing and aerodynamic friction, but mostly from the back emf of the DC drive motor. The equations of motion are

$$J\ddot{\theta} + B\dot{\theta} = T_c,$$

where T_c is the torque from the drive motor. Assume that

$$J = 600,000 \text{ kg}\cdot\text{m}^2 \quad B = 20,000 \text{ N}\cdot\text{m}\cdot\text{sec}.$$

(a) Find the transfer function between the applied torque T_c and the antenna angle θ.

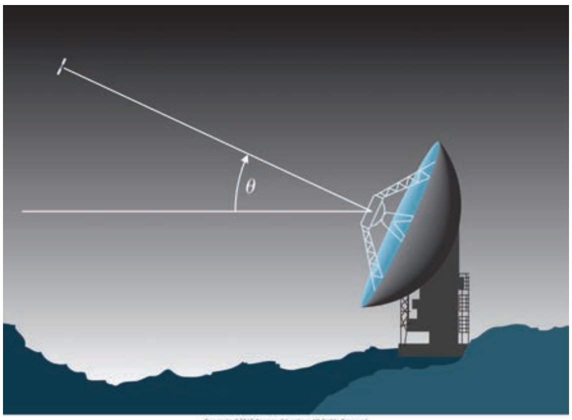
(b) Suppose the applied torque is computed so that θ tracks a reference command θ_r according to the feedback law

$$T_c = K(\theta_r - \theta),$$

where K is the feedback gain. Find the transfer function between θ_r and θ.

(c) What is the maximum value of K that can be used if you wish to have an overshoot $M_p < 10\%$?

(d) What values of K will provide a rise time of less than 80 sec? (Ignore the M_p constraint.)



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Figs 3.60 and 3.61

55. A positional servomechanism is characterized by an open-loop transfer function:

$$G(s)H(s) = \frac{k(s+2)}{s(s-1)}.$$

Determine the value of the gain k when (a) the damping ratio is 0.5 and (b) the closed-loop system has two roots on the $j\omega$ axis.

58. Consider the system shown in Fig. 3.67.

    (a) Compute the closed-loop characteristic equation.

    (b) For what values of (T, A) is the system stable? *Hint*: An approximate answer may be found using

    $$e^{-Ts} \cong 1 - Ts$$

    or

    $$e^{-Ts} \cong \frac{1 - \frac{T}{2}s}{1 + \frac{T}{2}s}$$

    for the pure delay. As an alternative, you could use the computer MATLAB (Simulink) to simulate the system or to find the roots of the system's characteristic equation for various values of T and A.

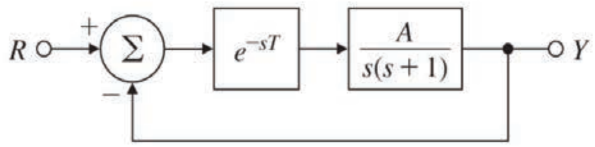
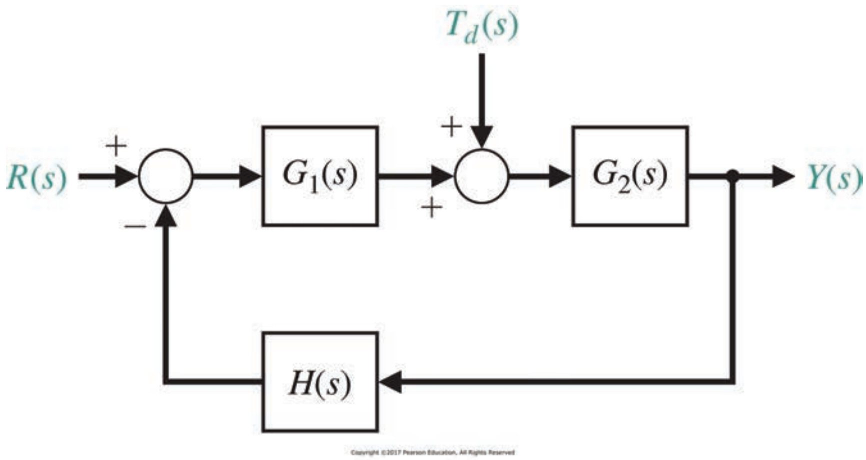
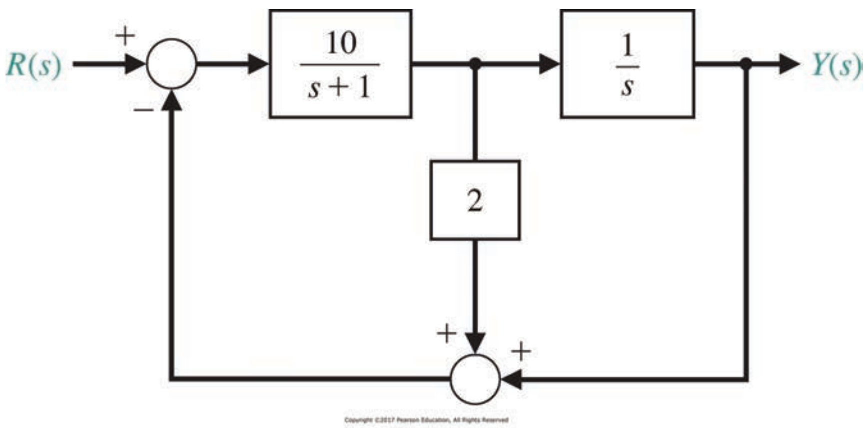


Figure 3.67: Control system for Problem 3.58

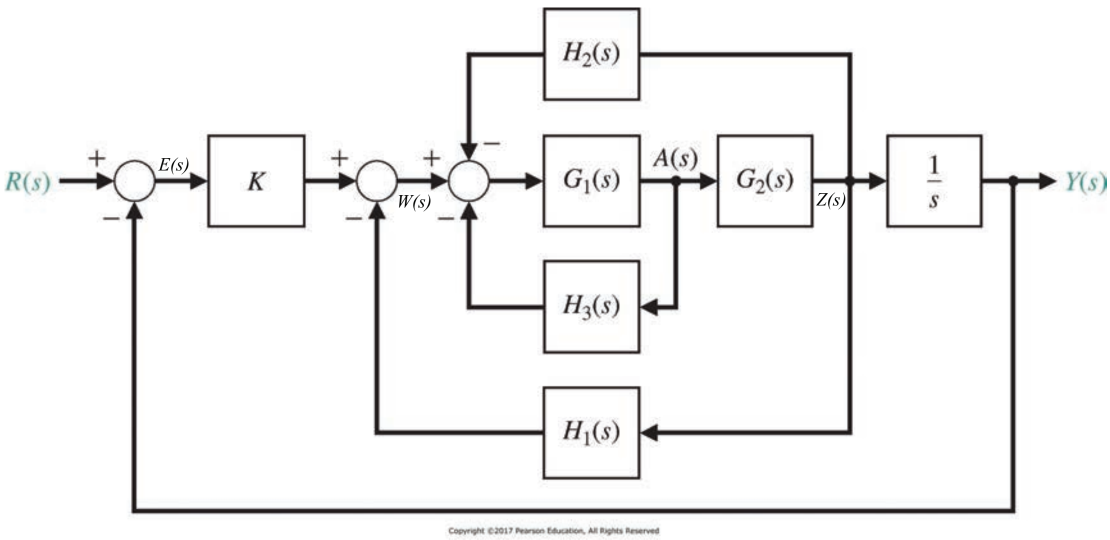
62. Determine the transfer functions $Y(s)/R(s)$ and $Y(s)/T_d(s)$ in the following block diagram:



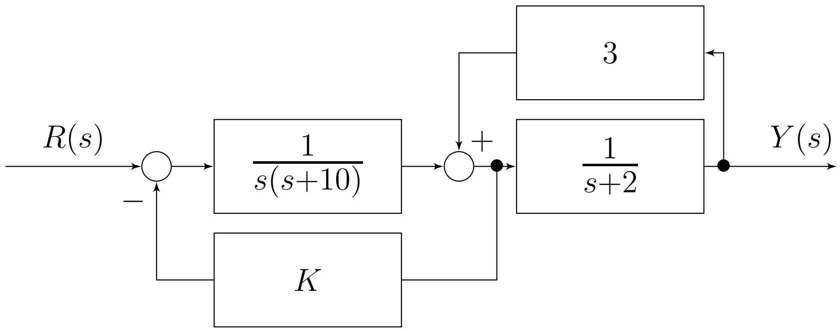
63. Determine the transfer function $Y(s)/R(s)$ in the following block diagram:



64. Determine the transfer function $Y(s)/R(s)$ in the following block diagram:



65. Consider the system given in the following block diagram:



a) Compute the transfer function between $R(s)$ and $Y(s)$.

b) For which values of K this transfer function is stable.