

Problem 1 solution

Solution

a)

$$H_c = \frac{Y}{Y_c} = \frac{G_r G_p}{1 + G_r G_p}$$

$$H_c = \frac{K \left(1 + \frac{1}{\tau_I s} + \tau_D s\right)}{s + 2 + K \left(1 + \frac{1}{\tau_I s} + \tau_D s\right)}$$

$$H_c = \frac{K(\tau_I \tau_D s^2 + \tau_I s + 1)}{(\tau_I + \tau_I \tau_D K)s^2 + (2\tau_I + K\tau_I)s + K}$$

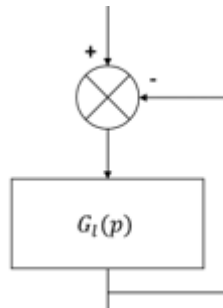
b) Fastest aperiodic poles are determined by

$$\Delta = (2\tau_I + K\tau_I)^2 - 4K(\tau_I + \tau_I \tau_D K) = 0$$

Since $\tau_I \neq 0$,

$$\tau_I(2 + K)^2 - 4K(1 + \tau_D K) = 0$$

c) First, let's simplify the looped G_l :



The equivalent transfer function $l(s)$ of this is:

$$H_l(s) = \frac{G_l(s)}{1 + G_l(s)}$$

$$H_l(s) = \frac{\frac{1}{s^2 + 2s - 1}}{1 + \frac{1}{s^2 + 2s - 1}}$$

$$H_l(s) = \frac{1}{s(s + 2)}$$

From there we calculate $\frac{Y}{D}$:

$$Y = H_l D + G_p G_r * (-Y)$$

$$\frac{Y}{D} = \frac{H_l}{1 + G_p G_r}$$

$$\frac{Y}{D} = \frac{1}{s(s + 2) + s(s + 2) * K \frac{\left(1 + \frac{1}{\tau_I s} + \tau_D s\right) * 1}{s + 2}}$$

$$\frac{Y}{D} = \frac{1}{s(s+2) + K\left(\tau_D s^2 + s + \frac{1}{\tau_I}\right)}$$

$$\frac{Y}{D} = \frac{1}{(K\tau_D + 1)s^2 + (2 + K)s + \frac{K}{\tau_I}}$$

d) The step disturbance $d(t) = D_0 e(t)$ at $t \rightarrow \infty$ can be calculated:

$$\lim_{t \rightarrow \infty} Y(t) = \lim_{s \rightarrow 0} s \cdot \frac{Y}{D} \cdot \frac{D_0}{s} = \frac{\tau_I}{K} D_0$$

e)

$$\frac{y_\infty - D_0}{D_0} \leq 0.1 \Leftrightarrow \frac{\tau_I}{K} \leq 1.1$$

f) The cheapest τ_I is the largest, and since $K = 10$ and $\tau_I \leq 1.1K$, we have $\tau_I = 11[s]$
From b),

$$\tau_I(2 + K)^2 - 4K(1 + \tau_D K) = 0$$

$$\tau_D = \tau_I \left(\frac{2 + K}{2K} \right)^2 - \frac{1}{K} = 3.86[s]$$

Problem 2 solution

We assume the derivatives to be zero.

In the Laplace domain, the equation becomes:

$$s^4 Y(s) + 2s^3 Y(s) + 2s^2 Y(s) + 4s Y(s) + 4Y(s) = 2sU(s) + U(s)$$

From there,

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2s + 1}{s^4 + 2s^3 + 2s^2 + 4s + 4}$$

2pts

The static gain K is given by:

$$K = \lim_{s \rightarrow 0} \frac{Y(s)}{U(s)}$$

Hence $K = \frac{1}{4}$

2pts

We can then use the Routh-Hurwitz criterion to study the stability of this system

s^4	1	2	4
s^3	2	4	
s^2	$0 \rightarrow \epsilon$	4	
s^1	$\frac{4\epsilon - 8}{\epsilon}$		
s^0	4		

4pts

If $\epsilon > 0$:	If $\epsilon < 0$:
$\frac{4\epsilon - 8}{\epsilon} < 0$	$\frac{4\epsilon - 8}{\epsilon} > 0$

1pt

In any case there are two sign changes, hence two poles with a positive real part.

2pts

If we loop the system with a P regulator $G_r(s) = K_r$, we get the new transfer function:

$$H(s) = \frac{G_r(s)G(s)}{1 + G_r(s)G(s)}$$

$$H(s) = \frac{(2s + 1)K_r}{s^4 + 2s^3 + 2s^2 + (4 + 2K_r)s + 4 + K_r}$$

2pts

We can use again the Routh-Hurwitz criterion to study the stability of the system:

s^4	1	2	$4 + K_r$
s^3	2	$4 + 2K_r$	
s^2	$-K_r$	$4 + K_r$	
s^1	$\frac{2}{K_r}(K_r^2 + 3K_r + 4)$		
s^0	$4 + K_r$		

4pts

From s^2 we get: $K_r < 0$

From s^1 we get: $K_r > 0$

From s^0 we get: $K_r > -4$

Hence, the system cannot be stabilized by any value of K_r

3pts

Problem 3 solution

a) We need to study $\frac{E(s)}{Y_c(s)} = \frac{(Y_c(s) - Y(s))}{Y_c(s)}$:

$$\frac{E(s)}{Y_c(s)} = \frac{Y_c(s) - Y(s)}{Y_c(s)} = 1 - G_{BF}(s)$$

$$\frac{E(s)}{Y_c(s)} = \frac{s^2 + 2\xi\omega_0 s + \omega_0^2 - K_d\omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2}$$

3pts

We want to determine the static error:

$$\lim_{t \rightarrow +\infty} e(t) = \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} s * \frac{E(s)}{Y_c(s)} * Y_c(s), \quad \text{with } Y_c(s) = \frac{1}{s} \text{ (step response)}$$

2pts

$$\lim_{t \rightarrow +\infty} e(t) = \lim_{s \rightarrow 0} \frac{E(s)}{Y_c(s)} = \frac{\omega_0^2 - K_d\omega_0^2}{\omega_0^2}$$

The condition for the static error to be nil is :

$$\lim_{t \rightarrow +\infty} e(t) = 0 \Leftrightarrow \frac{\omega_0^2 - K_d\omega_0^2}{\omega_0^2} = 0$$

3pts

$$\lim_{t \rightarrow +\infty} e(t) = 0 \Leftrightarrow K_d = 1$$

b)

$$G_{BF}(s) = \frac{G_r(s)G_p(s)}{1 + G_r(s)G_p(s)}$$

$$G_r(s) = \frac{G_{BF}(s)}{G_p(s) - G_p(s)G_{BF}(s)}$$

3pts

$$G_r(s) = \frac{s+1}{Ks}$$

From the course (p200, 5.9.5) we get:

$$G_r(s) = \frac{\tau_1\tau_2s^2 + (\tau_1 + \tau_2)s + 1}{K\tau_{BF}s}$$

$$\begin{cases} \tau_{BF} = 1 \\ \tau_1 + \tau_2 = 1 \\ \tau_1\tau_2 = 0 \end{cases} \Leftrightarrow \begin{cases} \tau_{BF} = 1 \\ \tau_1 = 0 \\ \tau_2 = 1 = \tau_I \end{cases}$$

And $K_r = \frac{\tau_1 + \tau_2}{K\tau_{BF}} = \frac{1}{K}$

3pts

c) We need to study $\frac{E(s)}{D(s)}$:

$$\frac{E(s)}{D(s)} = \frac{-G_l(s)}{1 + G_p(s)G_r(s)}$$

$$\frac{E(s)}{D(s)} = \frac{-1}{s^2(s+1) \left(1 + \frac{K}{(s+1)(s+1.4)} \cdot \frac{s+1}{Ks} \right)}$$

3pts

$$\frac{E(s)}{D(s)} = \frac{-(s+1.4)}{s(s+1)(s^2 + 1.4s + 1)}$$

We want to determine the static error in perturbation:

$$\lim_{t \rightarrow +\infty} e(t) = \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} s * \frac{E(s)}{D(s)} * D(s), \quad \text{with } D(s) = \frac{1}{s} \text{ (step perturbation)}$$

$$\lim_{t \rightarrow +\infty} e(t) = \lim_{s \rightarrow 0^+} \frac{E(s)}{D(s)} = -\infty$$

The corrector in b) cannot eliminate the influence of the perturbation. Since the perturbation has a double integrator, it would take at least a double integrator to rectify the perturbation to a non-divergent error, and a triple integrator to get it to 0. Other means of control might be considered at that point, at the level of the perturbation.

3pts

Problem 4 solution

ME-323 Commande de procédés

February 2nd, 2018

Examen

- 1) (20 points) Un système dynamique est donné par sa réponse indicielle :

$$y(t) = 0.25 - 0.25e^{-t} \cos \sqrt{3}t + 0.25\sqrt{3}e^{-t} \sin \sqrt{3}t, \quad t \geq 0$$

- Calculer la fonction de transfert de ce système.
- Évaluer le gain statique, la fréquence naturelle, l'amortissement, et les constantes de temps de ce système.
- Calculer et dessiner la réponse impulsionnelle du système.

Solution (si nécessaire, continuez au verso) :

a) Fonction de transfert : $G(s) = \frac{Y(s)}{U(s)}$

10 points

$$Y(s) = 0.25 \cdot \frac{1}{s} - 0.25 \frac{s+1}{(s+1)^2+3} + 0.25\sqrt{3} \frac{\sqrt{3}}{(s+1)^2+3} = 0.25 \left(\frac{1}{s} - \frac{s+1}{(s+1)^2+3} + \frac{3}{(s+1)^2+3} \right) =$$

$$= 0.25 \left(\frac{s^2+2s+4 - s^2 - s + 3s}{s((s+1)^2+3)} \right) = \frac{s+4}{s((s+1)^2+3)}$$

$$U(s) = \frac{1}{s} \quad (\text{indicielle}) \Rightarrow G(s) = \frac{Y(s)}{U(s)} = \frac{s(s+1)}{s((s+1)^2+3)} = \frac{s+1}{(s+1)^2+3}$$

b) Gain statique : $K = \lim_{s \rightarrow 0} G(s) = \frac{1}{4}$

5 points

* Forme canonique $G(s) : \frac{\kappa(\zeta s+1)}{s^2+2\zeta\omega_0 s+\omega_0^2}$

$\Rightarrow$ Fréquence : $\omega_0 = 2$

Amortissement : $\zeta = 0.5$

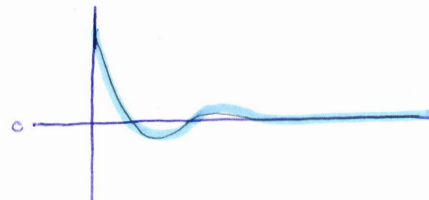
Const. temps : $\tau = \frac{1}{\omega_0} = \frac{1}{2}$

c) $U(s) = 1$ (impulsionnelle).

5 points

$G(s) = \frac{Y(s)}{U(s)} ; Y(s) = G(s) \cdot U(s) = G(s)$

$y(t) = \mathcal{L}^{-1}[G(s)] = \mathcal{L}^{-1}\left[\frac{s+1}{(s+1)^2+3}\right] = e^{-t} \cos(\sqrt{3}t)$



Etudiant:

Sciper no:

Problem 5 solution

a)

b) Boucle secondaire

$$\frac{T_w(s)}{T_{w,c}(s)} = \frac{R_2(s) G_2(s)}{1 + R_2(s) G_2(s)} = \frac{K_{R,2} \frac{0.2}{5s+1}}{1 + K_{R,2} \frac{0.2}{5s+1}}$$

$$= \frac{0.2 K_{R,2}}{5s+1 + 0.2 K_{R,2}} = \frac{\frac{0.2 K_{R,2}}{1+0.2 K_{R,2}}}{\frac{5}{1+0.2 K_{R,2}} s + 1} = \frac{K_2}{\tau_{BF,2} s + 1}$$

$\tau_{BF,2} = \frac{5}{1+0.2 K_{R,2}} \stackrel{!}{=} 1 \rightarrow \underline{K_{R,2} = 20}$

$K_2 = \frac{0.2 \times 20}{1+0.2 \times 20} = \frac{4}{5} = 0.8$

Boucle primaire

$$\frac{T(s)}{T_{w,c}(s)} = \frac{T(s)}{T_w(s)} \frac{T_w(s)}{T_{w,c}(s)} = \frac{0.5 e^{-s}}{10s+1} \cdot \frac{0.8}{s+1}$$

$$\approx \frac{0.4}{10s+1} e^{-2s}$$

Régulateur PI selon ZN:

$$K_{R,1} = 0.9 \frac{\tau}{\phi K} = 0.9 \frac{10}{2 \times 0.4} = \underline{11.25}$$

(Si nécessaire, continuez au verso)

$$\tau_{I,1} = 3.33 \theta = \underline{6.66}$$

c) Oui. Si $K_{R,2} \nearrow$, alors $K_2 \nearrow$ et $K \nearrow$

Pour $K_{R,1}$ donné, un grand K déstabilisera le système commandé.