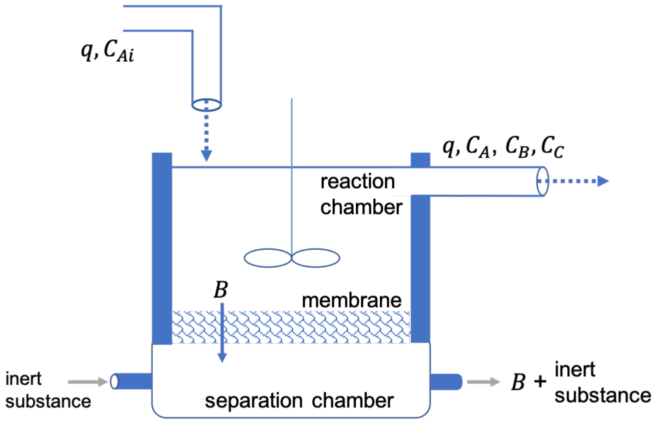
Exam

1. (30 points) A reversible reaction $2A \leftrightarrow B + C$ takes place in a membrane reactor shown below. The reactor consists of a well-mixed reaction chamber and a separation chamber separated by a membrane. The membrane is permeable to compound B, whereas compounds A and C cannot pass through. Compound B diffuses out of the reaction chamber through the membrane and is removed from the separation chamber by a stream of inert substance. The diffusion rate of B through the membrane is $k_m A_m C_B$ $[\frac{kmol}{min}]$, where A_m is the effective area of the membrane. Compound A is pumped into the reactor with the flow rate q and the concentration C_{Ai}. C_A, C_B, and C_C denote the concentrations of A, B, and C in the reactor. Despite the diffusion of B through the membrane, the exit and feed flow rates are assumed equal because B is a small molecule, i.e., it is assumed that the reactor volume is constant. The temperature in the reactor is also constant. The reaction follows the mass action kinetics with the rate constants $k_1 = 0.5\ \frac{m^3}{kmol \cdot min}$ (in the forward direction) and $k_2 = 2.5\ \frac{m^3}{kmol \cdot min}$ (in the reverse direction).



For the parameter values: reactor volume $V = 3\ m^3$, feed flow rate $q = 0.25\ m^3/min$, mass transfer coefficient $k_m = 1.5\ m/min$, and input feed concentration $C_{Ai} = 10\ kmol/m^3$:

a) Write the mass balances for compounds A and B.

b) Knowing that the effective membrane area at the steady-state is $A_m^{ss} = 4\ m^2$, and that the steady-state concentration of compound A is five times the one of B, i.e., $C_A^{ss} = 5C_B^{ss}$, determine the steady-state concentrations of A, B, and C in the reaction chamber.

c) The membrane degrades gradually and its effective area, A_m, changes in time. Derive the dynamic equations for deviations of concentrations C_A and C_B by linearizing the system of equations from a) around the steady-state derived in b).

d) Having the matrices in the states-space form, discuss how would you express in the Laplace domain the dependency of C_A and C_B on A_m. No explicit calculation is needed.

2. (25 points) A dynamical system response to the input signal

$$u(t) = \begin{cases} 4t & 0 \le t < 1.25 \\ 5 & t \ge 1.25 \end{cases}$$

is given by the following expression:

$$y(t) = t - 0.5e^{-2t}\sin 2t - (t - 1.25)\epsilon(t - 1.25) + 0.5e^{-2(t-1.25)}\sin\big(2(t - 1.25)\big)\,\epsilon(t - 1.25), t \ge 0$$

where $\epsilon(t)$ is the Heaviside function, i.e., $\epsilon(t) = 1,\ t \ge 0$.

a) Calculate the transfer function of the system.

b) Calculate the static gain K_S, the natural frequency ω_0, the damping factor ζ, and the time constant(s) τ of the system.

c) Calculate and sketch the impulse response of this system.

d) If the system from a) is controlled by a proportional controller with $K_R > 0$, find the values of K_R for which the resulting closed-loop system is critically damped ($\zeta = 1$), underdamped ($\zeta < 1$), and overdamped ($\zeta > 1$).

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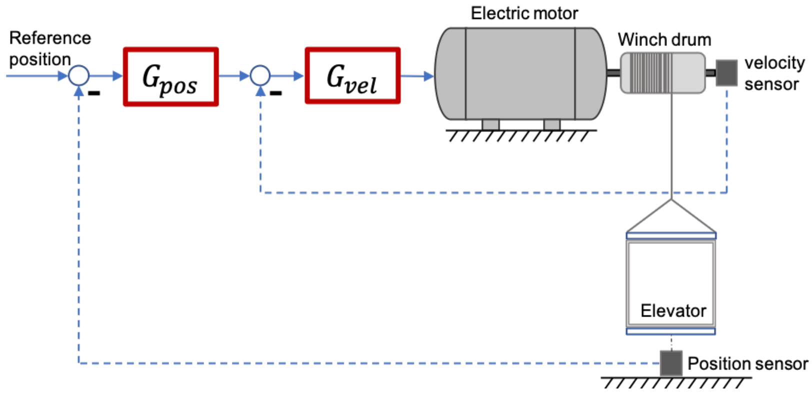
3. **(15 points)** The controller system for precise positioning of an elevator through cascaded velocity and position control is depicted below.



A control engineer has identified the following transfer functions in the system: $\frac{Y_{el}(s)}{\Omega(s)} = \frac{0.5}{s}$ and $\frac{\Omega(s)}{V_{em}(s)} = \frac{K_M/((sL+R)(sJ+B))}{1+K_M K_V/((sL+R)(sJ+B))}$, where Y_{el} denotes the position of the elevator, V_{em} the voltage applied at the input of the electric motor, and Ω is the angular velocity of the motor measured by the velocity sensor. Values of the parameters: $K_M = 1\ Nm/A,\ K_V = 1\ V/\frac{rad}{s}$, $L = 0.01\ H$, $R = 10\ ohm$, $J = 0.1\ Nm/\left(\frac{rad}{s}\right)^2$, and $B = 0.25\ Nm/\frac{rad}{s}$.

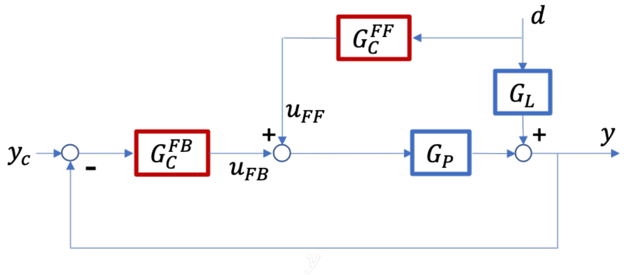
a) Identify the variables in the system (inputs, outputs) and discuss potential disturbances. Draw the block scheme of the control system. Discuss the dynamical properties of the inner and outer loops and the complexity of the corresponding controllers briefly.

b) Design the controllers for the cascade scheme with the following requirements

   i) Use the Ziegler-Nichols method to design a PI controller, G_{vel}, for the inner loop.

   ii) What is the simplest controller, G_{pos}, in the PID-form (P or PD or PI or PID) for the outer loop that ensures the zero steady-state error of a step response? Explain your answer. How would you choose the parameters of that controller? Explain in details without explicit calculations.

4. **(15 points)** Consider a dynamical system shown below



with the following transfer functions

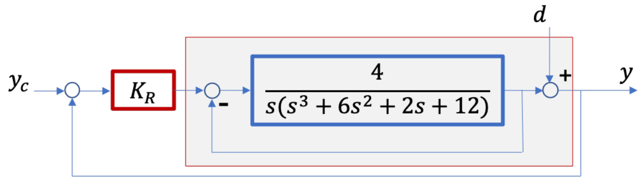
$$G_P(s) = \frac{0.5(s-0.25)}{(s+0.25)}, \qquad G_L(s) = \frac{0.1}{(s+0.5)}$$

a) Design a feedforward controller for this system using the knowledge of the transfer functions $G_P(s)$ and $G_L(s)$. Is the designed controller physically realizable?

b) For a step disturbance signal $d(t) = 5,\ t \geq 0$, compute the signal $u_{FF}(t)$, generated at the output of the feedforward controller computed at a). Discuss the obtained results.

c) Propose a lead-lag controller to eliminate the disturbance from b). Observe that the gain of the lead-lag controller, K_{L-L}, should be such that the disturbance is eliminated at the steady-state.

5. (15 points) Consider the control system depicted below.



a) Determine the stability of the system to be controlled (hatched in grey). If the system is unstable, determine and discuss how many unstable poles it possesses.

b) Find the range of K_R values that ensure the stability of the closed-loop system. Discuss the obtained results from the perspective of practical implementation.

c) Determine the steady-state error $y(\infty) - y_c(\infty)$ for the K_R values determined in b). Discuss the obtained result.

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