Control Systems I

Introduction

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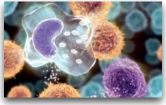
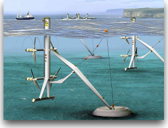
Laboratoire d'Automatique

Control Systems I

Prof. Colin Jones









Make things that **change with time** do what we want them to do

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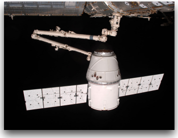
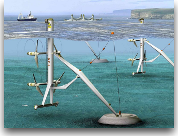


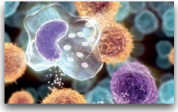
Most engineered systems **require controllers to function**





Controllers can provide **optimal performance**





Analysis and understanding of dynamic systems

A controller is anything that **senses** the environment, takes **decisions**, and **modifies the environment** in order meet some **objective**.

Robot Quadrotors Perform James Bond Theme

GRASP Lab, University of Pennsylvania

Components of a Control System

Sensor Measure the world

Actuator Effect the world

System The object we're trying to control

Controller Takes decisions based on

- Measurements
- Knowledge of how the system works

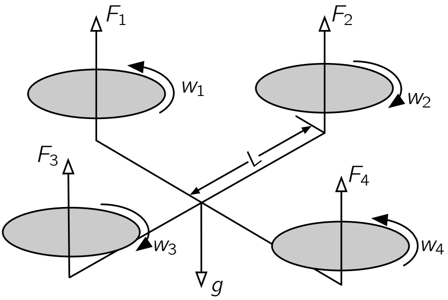
A controller is anything that **senses** the environment, takes **decisions**, and **modifies the environment** in order meet some **objective**.

Note: Controller doesn't have to be a 'computer', or an electronic circuit

Example: Autonomous Quadrocopter flight

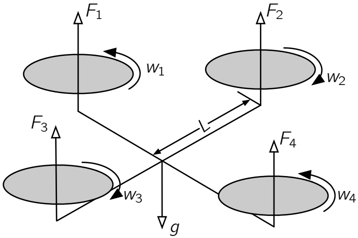
- Highly agile due to fast rotational dynamics
- High thrust-to-weight ratio allows for large translational accelerations
- Motion control by altering rotation rate and/or pitch of the rotors
- High thrust motors enable high performance control

How a Quad Works



- We can set the speed of the propellers (our inputs)
- Our goal is to control the pitch, roll and altitude

How a Quad Works



Force is quadratic in propeller speed:

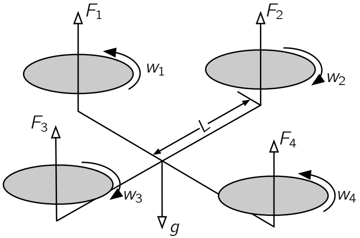
$$F_i(t) = k_F w_i(t)^2$$

Moment is quadratic in prop speed:

$$M_i(t) = k_M w_i(t)^2$$

Vertical force:	$F(t) = F_1(t) + F_2(t) + F_3(t) + F_4(t)$
Roll moment:	$M_\alpha(t) = L(F_1(t) - F_4(t))$
Pitch moment:	$M_\beta(t) = L(F_2(t) - F_3(t))$
Rotation:	$M_\gamma(t) = M_1(t) + M_2(t) + M_3(t) + M_4(t)$

How a Quad Works



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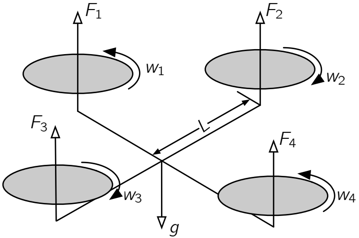
Vertical force:
Roll moment:
Pitch moment:
Rotation:

$$\begin{pmatrix} F(t) \\ M_\alpha(t) \\ M_\beta(t) \\ M_\gamma(t) \end{pmatrix} = \begin{bmatrix} k_F & k_F & k_F & k_F \\ Lk_F & 0 & 0 & -Lk_F \\ 0 & Lk_F & -Lk_F & 0 \\ k_M & k_M & k_M & k_M \end{bmatrix} \begin{pmatrix} w_1(t)^2 \\ w_2(t)^2 \\ w_3(t)^2 \\ w_4(t)^2 \end{pmatrix}$$

- We have four degrees of freedom and four forces / moments
- Can set the forces / moments as we like - these are our inputs

Quad Control

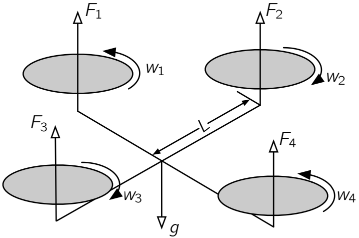


$$\text{Altitude:} \qquad m\ddot{z}(t) = \underbrace{-mg}_{\text{Gravity}} + \underbrace{F(t)}_{\text{Thrust of propellers}}$$

$$\text{Hold altitude at } z_c\text{:} \qquad F(t) = K(z_c - z(t))$$

$$\text{Resulting system:} \qquad m\ddot{z}(t) = -mg + K(z_c - z(t))$$

Quad Control



Roll and pitch:

$$I_\alpha \ddot{\alpha}(t) = M_\alpha(t)$$

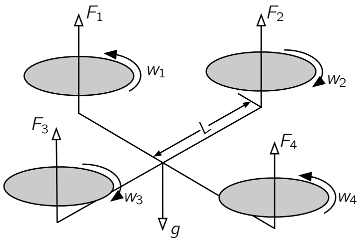
Hold attitude at α_c, β_c:

$$M_\alpha(t) = K_\alpha(\alpha_c - \alpha(t))$$

Resulting system

$$I_\alpha \ddot{\alpha}(t) = K_\alpha(\alpha_c - \alpha(t))$$

Quad Control



Yaw:

$$I_\gamma \ddot{\gamma}(t) = M_\gamma(t)$$

Keep yaw at zero:

$$M_\gamma(t) = -K_\gamma \gamma(t) - D_\gamma \dot{\gamma}(t)$$

Resulting system

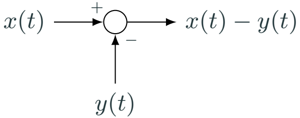
$$I_\gamma \ddot{\gamma}(t) = -K_\gamma \gamma(t) - D_\gamma \dot{\gamma}(t)$$

Example: Autonomous Quadrocopter flight

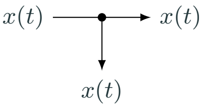
Demo movie

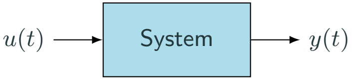
Lexus & Kmel robotics

Summation



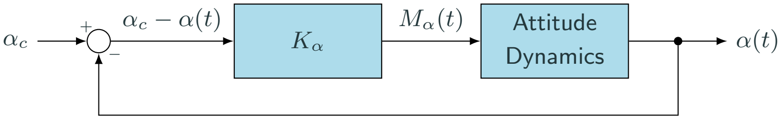
Bifurcation





Enforces a dynamic constraint between the output $y(t)$ and the input $u(t)$
e.g. $\ddot{y}(t) + \alpha\dot{y}(t) - \ddot{u}(t) + u(t) = 0$

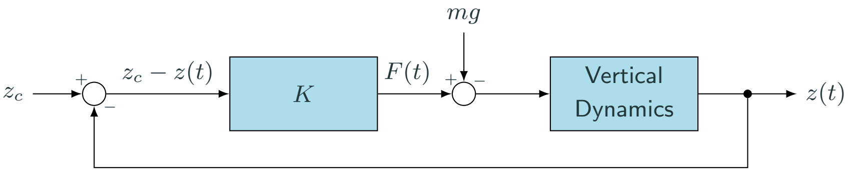
Block Diagram of Attitude Controllers



- Reference α_c
- Error $\alpha_c - \alpha(t)$
- Input $M_\alpha(t)$
- Output $\alpha(t)$
- Controller K_α
- System $I_\alpha \ddot{\alpha}(t) = M_\alpha(t)$

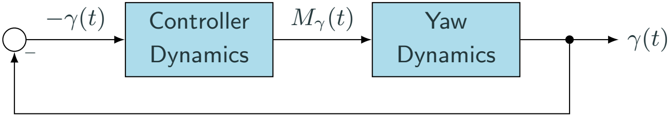
Goal: **Track** reference α_c

Block Diagram of Altitude Controller



- Disturbance g

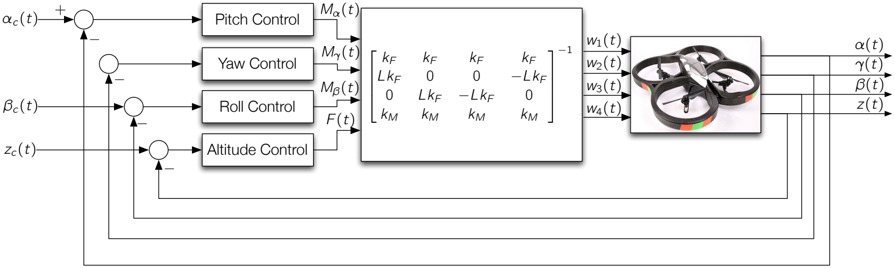
Goal: **Track** reference z_c and **reject** disturbance mg



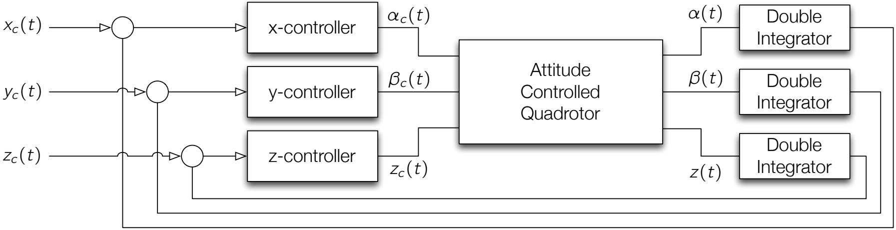
- Controller dynamics: $M_\gamma(t) = -K_\gamma \gamma(t) - D_\gamma \dot{\gamma}(t)$

Goal: **Regulate** the yaw

Cascade Control



Cascade Control

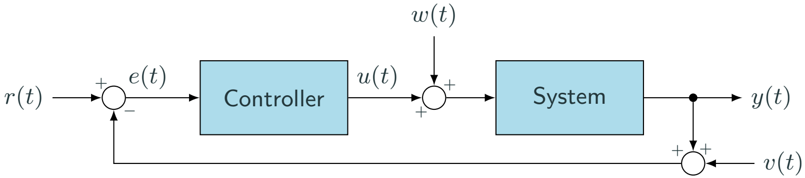


Possibly lots more loops

- Collision avoidance
- Trajectory planning
- Mission planning
- etc

Why?

- Inner loops make the system **predictable** and **simple**
- Conceptually simpler

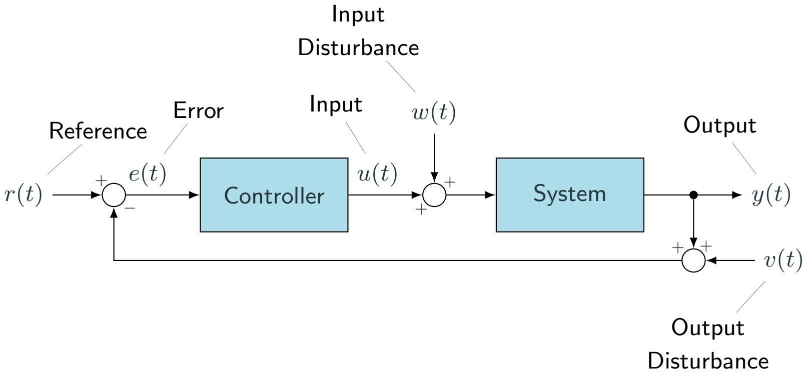


Goal: Make $y(t) = r(t)$, no matter what $w(t)$, or $v(t)$ are

If $r(t)$ is...

- zero, we're doing **regulation**
- time-varying, we're doing **servoing / tracking**

Canonical Block Diagram



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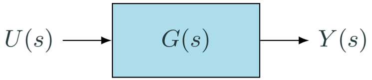
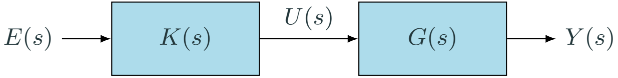
Nomenclature

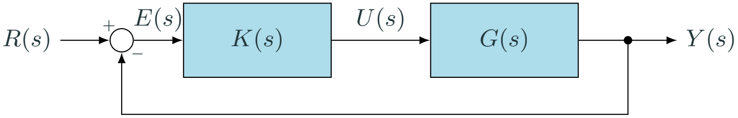
The **system**:



The **open-loop system** or **loop gain**:



The **closed-loop system**:



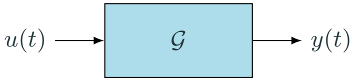
Quick Review of Systèmes Dynamique

More complete review on Moodle

What is a System?

A dynamic system transforms an input signal $u(t)$ into an output signal $y(t)$.

$$y = \mathcal{G}(u)$$



We care about LTI systems

Linear $\mathcal{G}(au_1 + bu_2) = a\mathcal{G}(u_1) + b\mathcal{G}(u_2)$

Causal $u(t) = 0$ for $t < 0$ implies $y(t) = 0$ for $t < 0$

Time-invariant $y(t) = \mathcal{G}(u(t))$ implies that $\mathcal{G}(u(t+T)) = y(t+T)$

Why are these types of systems important?

1. We can predict their behaviour from data easily

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Impulse Response

The impulse response $g(t)$ is defined as the output of the system in response to a dirac delta function at time $t = 0$:

$$g(t) := \mathcal{G}(\delta(t))$$

1. We can predict their behaviour from data easily

Impulse Response

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Theorem : Response of an LTI System

The output of an LTI system in response to an input signal $u(t)$ is

$$\mathcal{G}(u) = g * u$$

where $y = g * u$ if

$$y(t) = \int_0^t u(\tau)g(t-\tau)d\tau$$

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If $\mathcal{G}$ is an LTI system, then the impulse response completely characterizes it.

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If $\mathcal{G}$ is an LTI system, then the impulse response completely characterizes it.

Key limitation: Most systems have an infinitely-long impulse response.

Why are these types of systems important?

1. We can predict their behaviour from data easily
2. We can store and manipulate complex systems

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Transfer Function

The **transfer function** of a system is the Laplace transform of its impulse response.

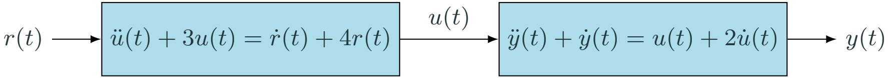
$$\mathcal{L}\{g(t)\} = G(s)$$

For LTI systems $G(s)$ is a rational polynomial function

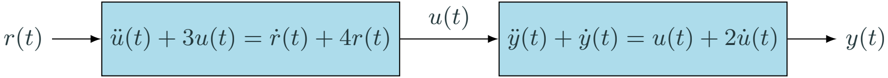
The point: Convolution becomes multiplication

$$y = g * u \qquad \Leftrightarrow \qquad Y(s) = G(s)U(s)$$

Manipulation of Simple Block Diagrams

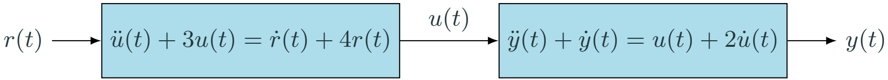


If we're given the reference function $r(t)$, what is $y(t)$?



$$\ddot{u}(t) + 3u(t) = \dot{r}(t) + 4r(t) \qquad \Rightarrow \qquad s^2U(s) + 3U(s) = sR(s) + 4R(s)$$

$$\ddot{y}(t) + \dot{y}(t) = u(t) + 2\dot{u}(t) \qquad \Rightarrow \qquad s^2Y(s) + sY(s) = U(s) + 2sU(s)$$



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Re-arranging gives:

$$U(s) = \frac{s+4}{s^2+3}R(s) \qquad\qquad Y(s) = \frac{1+2s}{s^2+s}U(s)$$

Manipulation of Simple Block Diagrams

$$r(t) \longrightarrow \boxed{\ddot{u}(t) + 3u(t) = \dot{r}(t) + 4r(t)} \xrightarrow{u(t)} \boxed{\ddot{y}(t) + \dot{y}(t) = u(t) + 2\dot{u}(t)} \longrightarrow y(t)$$

$$\begin{aligned} \ddot{u}(t) + 3u(t) = \dot{r}(t) + 4r(t) \qquad &\Rightarrow \qquad s^2U(s) + 3U(s) = sR(s) + 4R(s) \\ \ddot{y}(t) + \dot{y}(t) = u(t) + 2\dot{u}(t) \qquad &\Rightarrow \qquad s^2Y(s) + sY(s) = U(s) + 2sU(s) \end{aligned}$$

Re-arranging gives:

$$U(s) = \frac{s+4}{s^2+3}R(s) \qquad\qquad Y(s) = \frac{1+2s}{s^2+s}U(s)$$

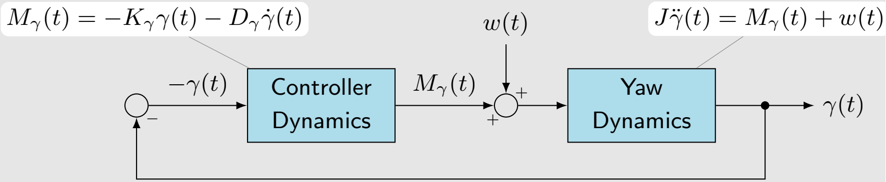
... and we can compute the impact of $r(t)$ on $y(t)$

$$Y(s) = \frac{1+2s}{s^2+s} \cdot \frac{s+4}{s^2+3}R(s)$$

Series connection of blocks (convolution) becomes multiplication!

$$R(s) \longrightarrow \boxed{\frac{1+2s}{s^2+s} \cdot \frac{s+4}{s^2+3}} \longrightarrow Y(s)$$

Compute response to a impulsive disturbance acting on the yaw system



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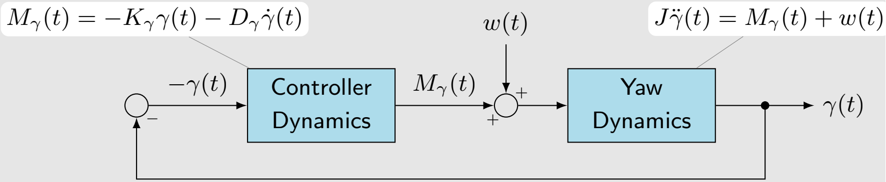
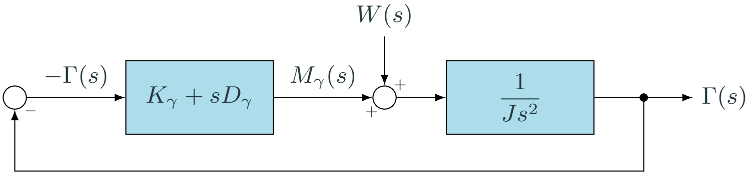
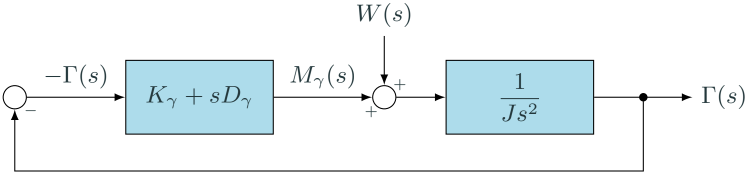




Example: System Response



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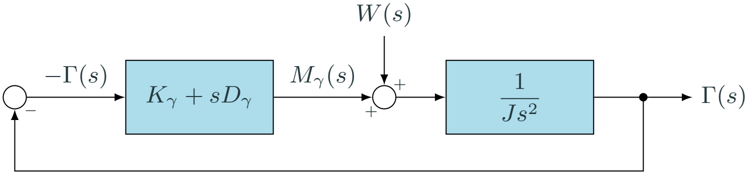
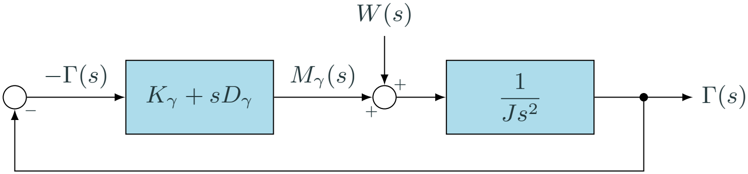


Start at the output and work backwards against the arrows

$$\Gamma = \frac{1}{Js^2}(W - (K_\gamma + sD_\gamma)\Gamma)$$

$$(Js^2 + sD_\gamma + K_\gamma)\Gamma = W$$



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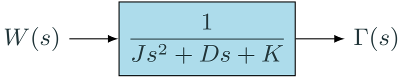
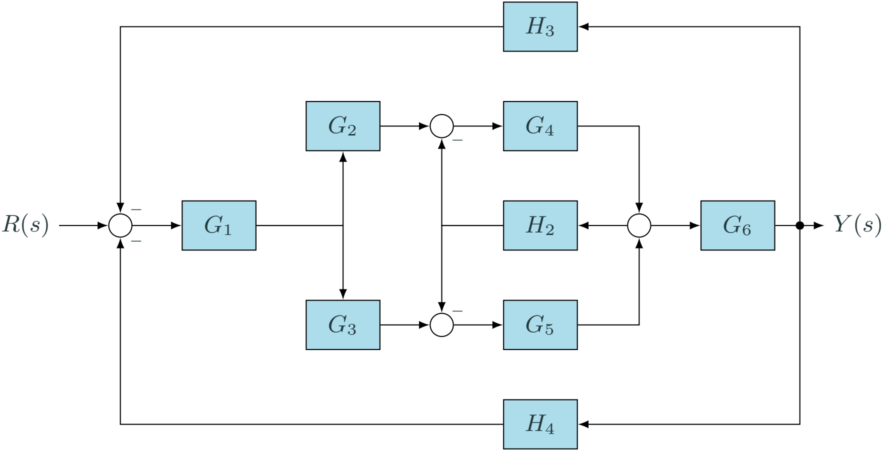
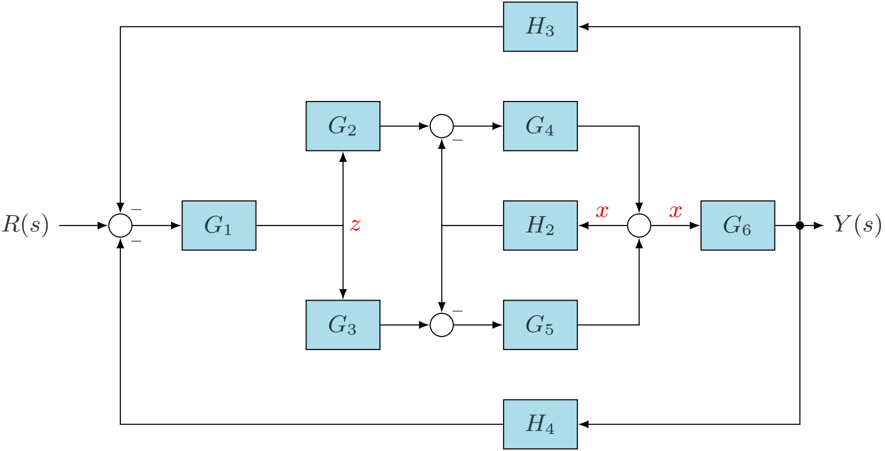


Where we recall that D sets the damping and K the response rate.

Example: Complex System



Example: Complex System



Add auxiliary variables for internal loops, and wherever convenient to simplify.

Example: Complex System

Start at the output and work back **against** the arrows.
A block is a multiplication, a summation is addition.

$$
\begin{aligned}
Y &= G_6 x \\
x &= G_4(G_2 z - H_2 x) + G_5(G_3 z - H_2 x) \\
z &= G_1(R - H_3 Y - H_4 Y)
\end{aligned}
$$

Solve for Y as a function of R

$$
\begin{aligned}
x &= (G_4G_2 + G_5G_3)z - (G_4H_2 + G_5H_2)x \\
(1 + G_4H_2 + G_5H_2)x &= (G_4G_2 + G_5G_3)z \\
x &= \frac{G_4G_2 + G_5G_3}{1 + G_4H_2 + G_5H_2} z \\
Y &= G_6 \frac{G_4G_2 + G_5G_3}{1 + G_4H_2 + G_5H_2} z
\end{aligned}
$$

Example: Complex System

$$Y = \underbrace{G_6 \frac{G_4 G_2 + G_5 G_3}{1 + G_4 H_2 + G_5 H_2}}_{Q} z \qquad\qquad z = G_1 R - (H_3 + H_4) Y$$

Solve to get the transfer function

$$\frac{Y}{R} = \frac{Q G_1}{1 + Q(H_3 + H_4)}$$

If we want to do more algebra, we can eliminate Q

$$\frac{Y}{R} = \frac{G_1 G_2 G_4 G_6 + G_1 G_3 G_5 G_6}{(G_4 + G_5) H_2 + G_2 G_4 G_6 H_3 + G_2 G_4 G_6 H_4 + G_3 G_5 G_6 H_3 + G_3 G_5 G_6 H_4 + 1}$$

Why are these types of systems important?

1. We can predict their behaviour from data easily
2. We can store and manipulate complex systems
3. We can shape system behaviour

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Time domain

- PID
- Model predictive control
- ...

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Frequency domain

- Loopshaping controllers
- $\mathcal{H}_\infty$ - robust optimal control
- ...

3. We can shape system behaviours

Time domain

- PID
- Model predictive control
- ...

Frequency domain

- Loopshaping controllers
- $\mathcal{H}_\infty$ - robust optimal control
- ...

Pole/zero domain

- Pole placement
- Linear quadratic regulation
- ...

3. We can shape system behaviours

Time domain

- PID
- Model predictive control
- ...

Frequency domain

- Loopshaping controllers
- $\mathcal{H}_\infty$ - robust optimal control
- ...

Pole/zero domain

- Pole placement
- Linear quadratic regulation
- ...

Many very well-established techniques that are proven and work well at large scales.

Key points to review

Please review:

- Computation of Laplace transforms
- Manipulation of block diagrams
- Inverse Laplace transforms
- System response to impulse, step, ramp, etc

Administration

Teachers

Professor
Colin Jones
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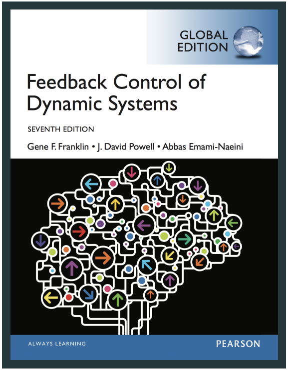
Reference Material

We will mostly follow the textbook:



- The sections of the text that we are covering will appear on Moodle
- Lecture notes and pre-recorded videos are on Moodle

You are responsible for the material in the text **and** in the lecture notes

Activities

1. Lectures
   - Two hours per week
   - Lectures are not recorded, but high-quality pre-recordings are on Moodle
2. TPs
   - Seven TPs done via a MOOC interface driving a physical device
   - Can do the TPs in-person **or** remotely
3. Exercises
   - Written / computer exercises
   - 13 exercise sets

$\boxed{\text{Detailed schedule on Moodle}}$

How to Get Help

In person During lectures, or during afternoon exercise / TP sessions

Ed Discussion Please post your questions publicly - others will benefit!

Recorded videos Lectures have been pre-recorded and are available on Moodle.

Grading and Exams

100% Final exam

- One question from the TPs (MOOC) worth 20%
- Questions based on the lectures / exercises worth 80%

Examples: Other Varieties of Control

Employee Scheduling : The Challenge

Too few salespeople

=

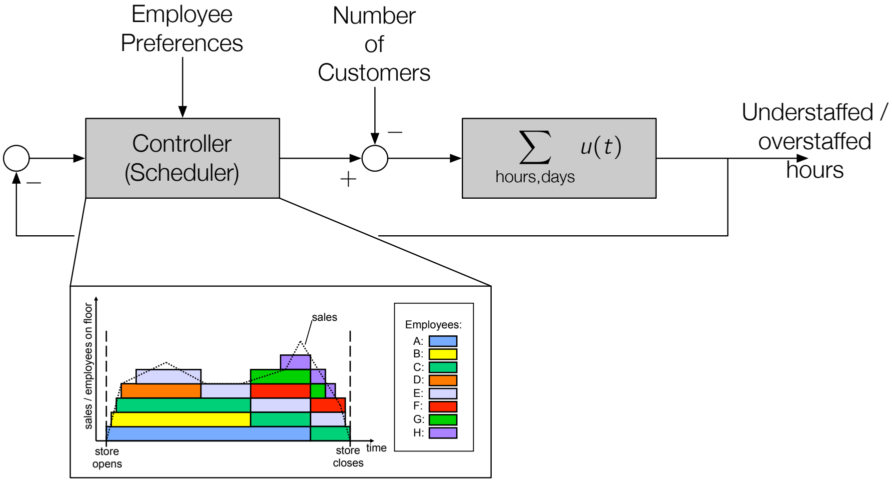
Unhappy customers / less sales

Too many salespeople

=

Excessive wages

What can control do?

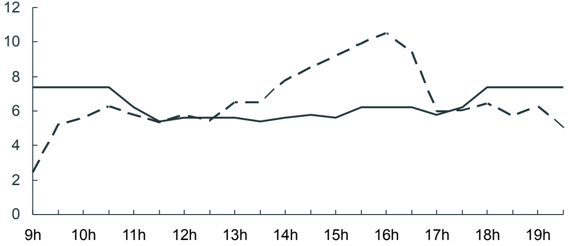

Employee Preferences
Number of Customers
Controller (Scheduler)
−
+
−
$\sum_{\text{hours,days}} u(t)$
Understaffed / overstaffed hours
sales
sales / employees on floor
store opens
store closes
time
Employees:
A:
B:
C:
D:
E:
F:
G:
H:

… ACHIEVES SAVINGS BY MATCHING RESOURCES TO DEMAND

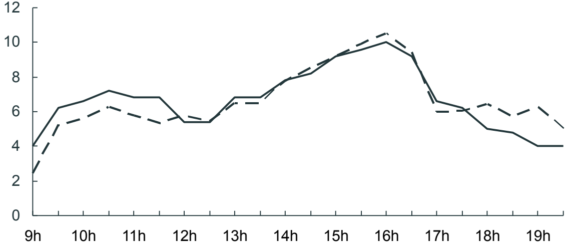
EXAMPLE

Average number of weekday staff*

Scheduled
Demanded



- Original scheme (manually scheduled, unoptimized)
- FTE hours per week
  - Excess scheduled: 65
  - Unmet demanded: 85
  - Mismatched: 150



- Optimized schedule** (automatically produced)
- FTE hours per week
  - Excess scheduled: 15
  - Unmet demanded: 39
  - Mismatched: 54
- Estimated savings of up to 46 overtime hours per week (~10% of total) needed to meet expected demand

* For retail store with 14 staff (11.5 FTEs)

** Sample optimized schedule provided by Apex Optimization GmbH

Example: ‘Fulfilment Centers’

Kiva systems

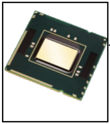
Sold to Amazon in March, 2012 for $775m USD

The Inerter in F1 Racing

Slides from Prof. Malcom Smith

Demand Response Slides

Control Applications at all Space and Time Scales



Computer control — ns

μs — Power systems

Traction control — ms

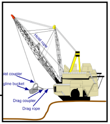
Seconds — Buildings

Refineries — Minutes

Hours — Nurse rostering

Train scheduling — Days

Weeks — Production planning



Summary

- Feedback control is everywhere
- It is used to:
  - Stabilize unstable systems
  - Make behaviors repeatable / predictable
  - Maximize performance
  - Understand what complex systems are doing