Control Systems : Set 8 : Loopshaping (4)

Prob 1 | For the closed-loop transfer function

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

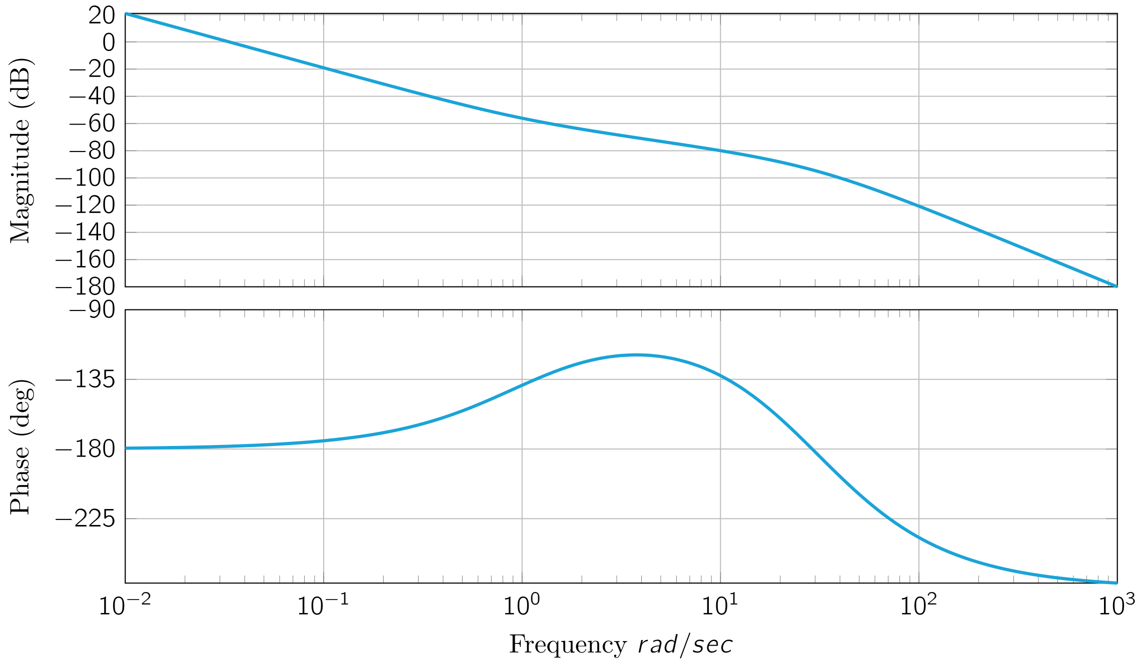
derive the following expression for the bandwidth ω_{BW} of $T(s)$ in terms of ω_n and ζ

$$\omega_{BW} = \omega_n\sqrt{1 - 2\zeta^2 + \sqrt{2 + 4\zeta^4 - 4\zeta^2}}$$

Assuming $\omega_n = 1$, use Matlab to plot ω_{BW} for $0 < \zeta < 1$.

Prob 2 | The Bode plot of the following system for $K = 1$ is given below

$$KG(s) = \frac{K(s+1)}{s^2(s+30)^2}$$



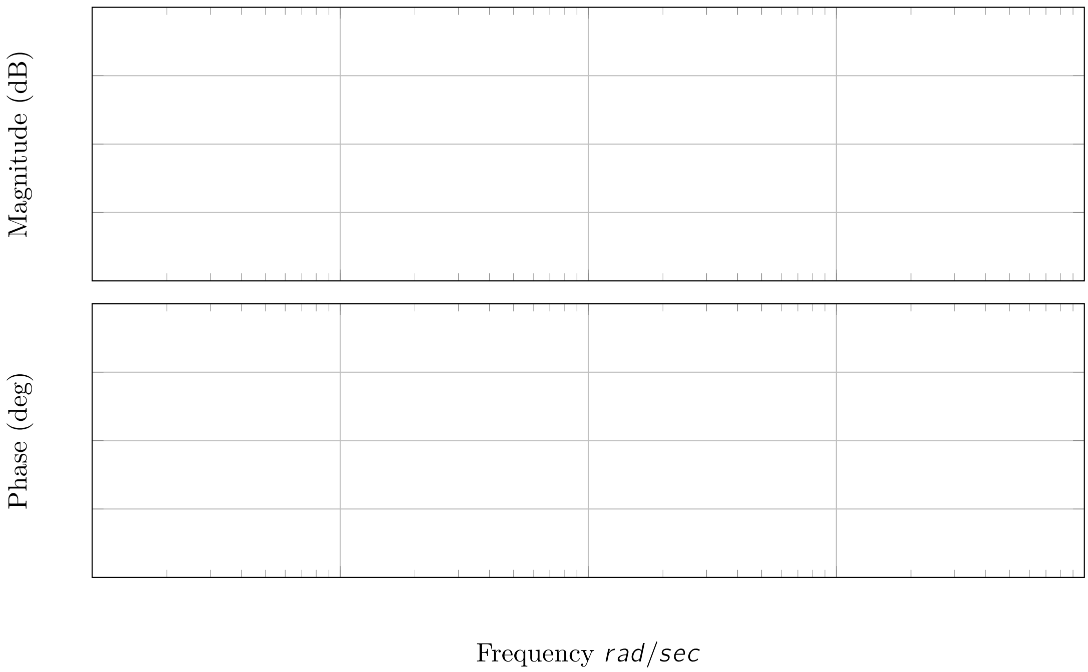
a) Determine the range of gains K that will yield a phase margin of at least $30°$.

b) What is the maximum possible closed-loop bandwidth that satisfies $PM \geq 30°$?

c) Use Matlab to confirm your finding.

Prob 3 | Design a lead compensator $D_c(s)$ with unity DC gain so that $PM \geq 40°$ using Bode plot sketches, then verify your design using Matlab. What is the approximate bandwidth of the system?

$$G(s) = \frac{5}{s(s+1)(s/5+1)}$$

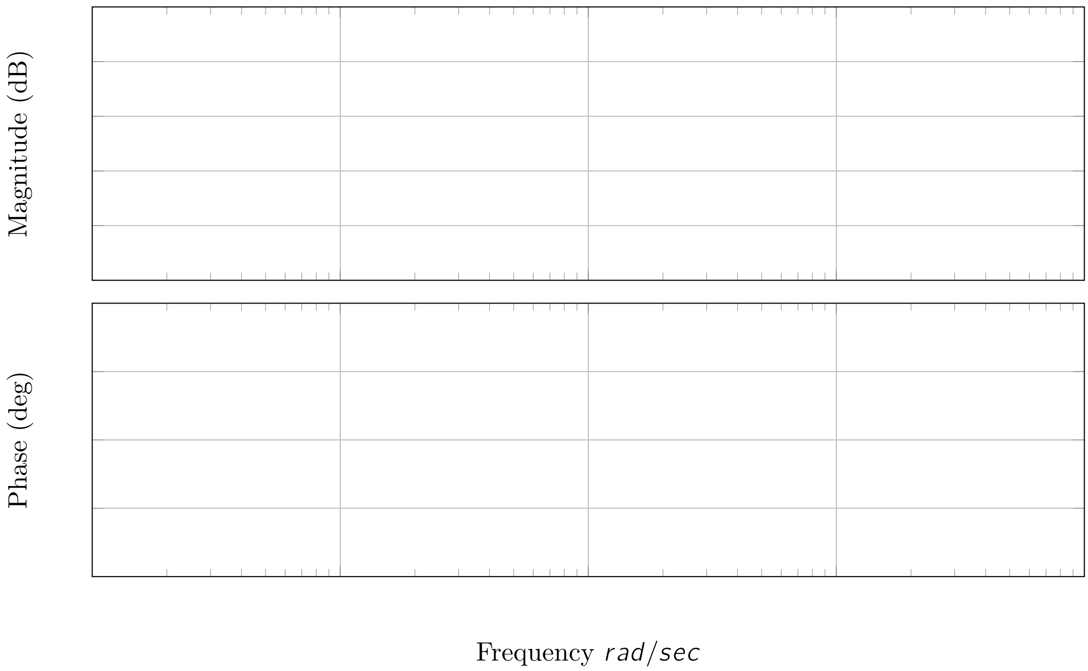


Prob 4 | The inverted pendulum has a transfer function given by

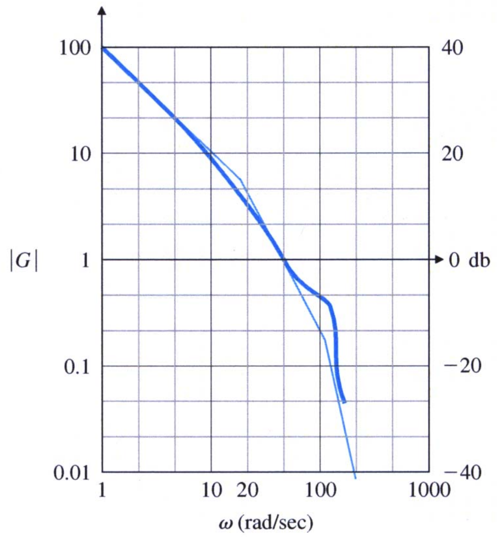
$$G(s) = \frac{1}{s^2 - 1}$$

a) Design a lead compensator to achieve a PM of 30° and a bandwidth around $1r/s$ using a Bode plot sketch, then verify and refine your design using Matlab.

b) Could you obtain the frequency response of this system experimentally?



Prob 5 | The frequency response of a plant in a unity-feedback configuration is sketched in the figure below. Assume that the plant is open-loop stable and minimum-phase.



a) What is the velocity constant K_v for the system as drawn?

b) What is the damping ratio of the complex poles at $\omega = 100$?

c) Estimate the PM of the system as drawn. (Hint: Bode phase-gain relationship)