Control Systems : Set 2 : PID (1)

Prob 1 | The dynamics of a DC-motor is described by the differential equation

$$\dot{y} + 60y = 600v_a - 1500w$$

where y is the motor speed, v_a is the armature voltage, and w is the load torque. Assume the armature voltage is computed using the PI control law

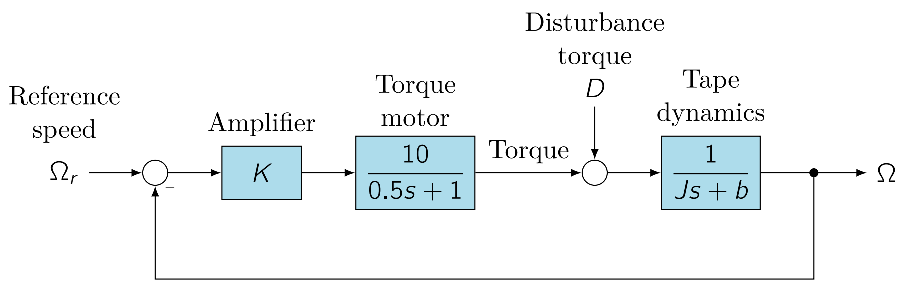
$$v_a = k_p e + k_I \int_0^t e dt$$

where $e = r - y$ for the reference speed r.

a) Compute the transfer function from W to Y as a function of k_p and k_I

b) Compute values for k_p and k_I so that the characteristic equation of the closed-loop system will have roots at $-60 \pm 60j$.

Prob 2 | A speed control system for a magnetic tape-drive is shown in the figure below, where the constants are $J = 0.10 kg \cdot m^2$ and $b = 1.00 N \cdot m \cdot sec$.

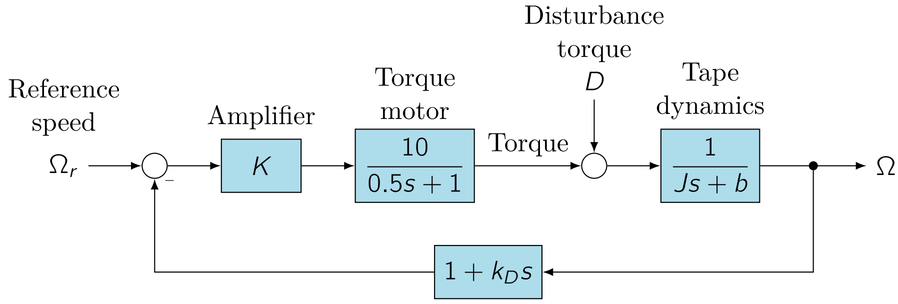


a) Assuming the reference is zero, what is the steady-state error due to a step disturbance torque of $1Nm$? What must the amplifier gain K be in order to achieve a steady-state error $e_{ss} \leq 0.01$rad/sec?

b) Give the damping ratio and the natural frequency of the closed-loop system from Ω_r to Ω, and sketch the time response of the output for a step reference input using the gain K computed in the previous part. Is this a good control system?

c) Give values for K and k_D for a PD controller which will meet the specifications of a 1% settling time of $t_s \leq 0.1$sec and an overshoot $M_p \leq 5\%$.

Note: Recall that we don't take the derivative of the reference, and so we place the derivative term in the feedback path as shown below.



d) How would the disturbance-induced steady-state error change with the new control scheme in the previous part? How could the steady-state error to a disturbance torque be eliminated entirely?

Prob 3 | A linear ODE model of a DC motor with negligible armature inductance and with a disturbance torque w is given by

$$\ddot{\theta} + a_1\dot{\theta} = b_0 v_a + c_0 w$$

where θ is the position of the motor and is measured in radians, v_a is the applied voltage in Volts, w is the load torque and a_1, b_0 and c_0 are motor-dependent constants.
With rotating potentiometers, it is possible to measure the positioning error between θ and the reference angle θ_r or $e = \theta_r - \theta$. With a tachometer we can measure the motor speed $\dot{\theta}$.

a) Draw a block diagram of the resulting feedback system showing both θ and $\dot{\theta}$ as variables in the diagram representing the motor.

b) Suppose the motor constants are $a_1 = 65$, $b_0 = 200$, and $c_0 = 10$. If there is no load torque ($w = 0$), what speed (in rpm) results from $v_a = 100V$?

c) Using the parameter values given in the previous part, consider the feedback controller on the error $e = \theta_r - \theta$ and the motor speed $\dot{\theta}$ in the form

$$v_a = k_p e - k_D\dot{\theta}$$

Select the controller parameters k_p and k_D such that the response to a step input in the reference has approximately 17% overshoot and settles within 5% of steady-state in less than 0.05 seconds, when the disturbance is zero $w = 0$.