Control Systems : Set 13 : LQR

Prob 1 | Consider the double integrator system $G(s) = \frac{1}{s^2}$ given in control canonical form

$$\dot{x} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

a) Give the optimal LQR controller $u = -Kx$ for the following weighting matrices

$$R = 1 \qquad\qquad Q = \begin{bmatrix} \alpha^2 - 2\beta & 0 \\ 0 & \beta^2 \end{bmatrix}$$

b) Give the characteristic equation for the closed-loop system

c) Choose an α and β such that the closed-loop system is critically damped, with a natural frequency of 5 rad/s.

Prob 2 | Consider the system

$$G(s) = \frac{\alpha}{s+1}$$

a) Give a state-space model in control canonical form

b) Design a state-feedback controller so that the closed-loop system has a pole at -2

c) Suppose that α changes to 2α, how does the closed-loop pole change? Is this result surprising? Why?

d) Design an estimator for the system so that the estimator pole is 5 times faster than the closed-loop pole.

e) Add a reference input to the controller so that the output tracks constant inputs in steady-state

f) Suppose that the controller was designed for α, but the real system has a value $\tilde{\alpha}$. Does your system track the reference?

   Hint: Compute the transfer function from the reference to the output for the closed-loop system, and then use the final value theorem.

g) Add an integrator to the system and repeat the controller and estimator design processes, setting both closed-loop poles to -2. Show that it tracks reference inputs even if the value of α used in the design is different from that of the real system.

   Note: Use Matlab with various values of α and $\tilde{\alpha}$ to verify the result with the dcgain function, or by plotting step responses, etc. If you do it with the final value theorem as in the last question, you'll have to compute a 3x3 matrix inverse. This can also be done with the Matlab symbolic toolbox if you prefer.