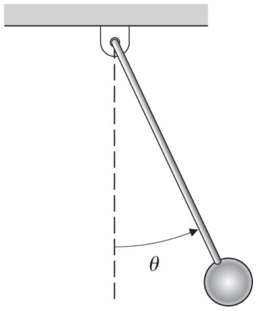
Control Systems : Set 12 : Statespace (3)

Prob 1 | The linearized equations of motion of the simple pendulum in the figure below are

$$\ddot{\theta} + \omega^2\theta = u$$



a) Write the equations of motion in state-space form.

b) Design an estimator (observer) that reconstructs the state of the pendulum given measurements of $\dot{\theta}$. Assume $\omega = 5$ rad/sec, and pick the estimator roots to be at $s = -10 \pm 10j$.

c) Write the transfer function of the estimator between the measured value of $\dot{\theta}$ and the estimated value of θ.

d) Design a controller (that is, determine the state feedback gain K) so that the roots of the closed-loop characteristic equation are at $s = -4 \pm 4j$.

Prob 2 | A certain process has the transfer function

$$G(s) = \frac{4}{s^2 - 4}$$

a) Find A, B and C for this system in observer canonical form

b) If $u = -Kx$, compute K so that the closed-loop control poles are located at $s = -2 \pm 2j$

c) Compute L so that the estimator-error poles are located at $s = -10 \pm 10j$

d) Give the transfer function of the resulting controller

e) What are the gain and phase margins of the controller and the given open-loop system?

Prob 3 | Consider the control of

$$G(s) = \frac{Y(s)}{U(s)} = \frac{10}{s(s+1)}$$

a) Let $y = x_1$ and $\dot{x}_1 = x_2$ and write the state equations for the system

b) Find K_1 and K_2 so that $u = -K_1x_1 - K_2x_2$ yields closed-loop poles with a natural frequency $\omega_n = 3$ and a damping ratio $\zeta = 0.5$

c) Design a state estimator for the system that yields estimator error poles with $\omega_{n1} = 15$ and $\zeta_1 = 0.5$

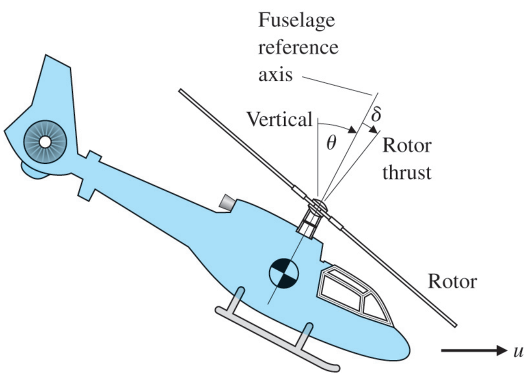
d) What is the transfer function of the controller obtained by combining parts (a) - (c)?

Prob 4 | The linearized longitudinal motion of a helicopter near hover (figure below) can be modeled by the normalized third-order system

$$\begin{bmatrix} \dot{q} \\ \dot{\theta} \\ \dot{u} \end{bmatrix} = \begin{bmatrix} -0.4 & 0 & -0.01 \\ 1 & 0 & 0 \\ -1.4 & 9.8 & -0.02 \end{bmatrix} \begin{bmatrix} q \\ \theta \\ u \end{bmatrix} + \begin{bmatrix} 6.3 \\ 0 \\ 9.8 \end{bmatrix} \delta$$

where

$$\begin{aligned} q &= \text{pitch rate} \\ \theta &= \text{pitch angle of fuselage} \\ u &= \text{horizontal velocity (standard aircraft notation)} \\ \delta &= \text{rotor tilt angle (control variable)} \end{aligned}$$



Suppose our sensor measures the horizontal velocity u as the output, that is $y = u$
Use Matlab to answer the following questions.

a) Design a state estimator with poles at -8 and $-4 \pm 4\sqrt{3}j$

b) Compute the compensator transfer function using control you design for this problem in exercise set 5, and the estimator designed above.

c) Draw Bode plots for the loop gain and the closed-loop system. What is the bandwidth, gain margin and phase margin?

Prob 5 | The linearized equations of motion for a satellite are

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3\omega^2 & 0 & 0 & 2\omega \\ 0 & 0 & 0 & 1 \\ 0 & -2\omega & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

The inputs u_1 and u_2 are the radial and tangential thrusts, the state variables x_1 and x_3 are the radial and angular deviations from the reference (circular) orbit, and the outputs y_1 and y_2 are the radial and angular measurements, respectively.

a) Show that the system is controllable using both control inputs

b) Show that the system is controllable using only a single input. Which one is it?

c) Show that the system is observable using both measurements.

d) Show that the system is observable using only one measurement. Which one it it?

Note that the definitions of controllability and observability matrices are valid for multiple inputs and outputs, and that full rank of these matrices is what's required for controllability and observability.

Prob 6 | Consider a system with the transfer function

$$G(s) = \frac{9}{s^2 - 9}$$

a) Find (A, B, C) for this system in observer canonical form

b) Is (A, B) controllable?

c) Compute K so that the closed-loop poles are assigned to $s = -3 \pm 3j$

d) Is the system observable?

e) Design an estimator with estimator poles at $s = -12 \pm 12j$

f) Suppose the system is modified to have a zero

$$G_1(s) = \frac{9(s+1)}{s^2 - 9}$$

Prove that if $u = -Kx + r$, there is a feedback gain K that makes the closed-loop system unobservable. (Again assume an observer canonical realization for $G_1(s)$.)