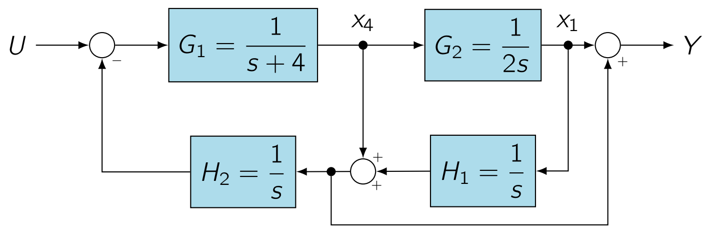
Control Systems : Set 10 : Statespace (1)

Prob 1 | Given the system

$$\dot{x} = \begin{bmatrix} -5 & 1 \\ -2 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

with zero initial conditions, find the steady-state value of x for a step input u.

Prob 2 | Consider the system shown below



a) Find the transfer function from U to Y

b) Write state equations for the system using the state variables indicated

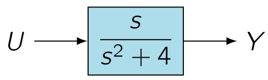
Prob 3 | Show that a transfer function is not changed by a linear transformation of the state.

Prob 4 | For each of the listed transfer functions, write the state equations in control canonical form. In each case draw a block diagram and give the appropriate expressions for A, B and C.

a) $G(s) = \dfrac{s^2 - 2}{s^2(s^2 - 1)}$

b) $G(s) = \dfrac{3s + 4}{s^2 + 2s + 2}$

Prob 5 | Consider the system in the figure below.



a) Write a set of equations that describes this system in the control canonical form as $\dot{x} = Ax + Bu$ and $y = Cx$

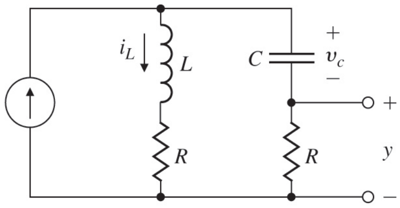
b) Design a control law of the form

$$u = -\begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

which will place the closed-loop poles at $s = -2 \pm 2j$

c) Check your answer with Matlab

Prob 6 | Consider the electric circuit shown in the figure below



a) Write the internal (state) equations for the circuit. The input $u(t)$ is a current, and the output y is a voltage. Let $x_1 = i_L$ and $x_2 = v_c$

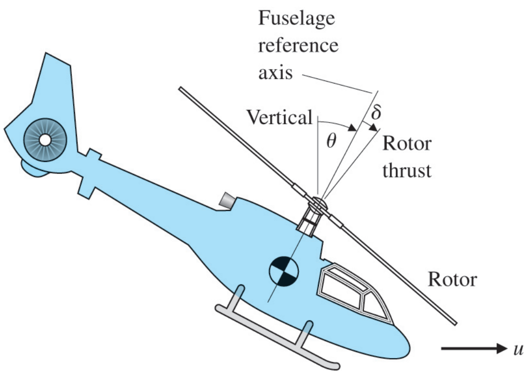
b) What condition(s) on R, L and C will guarantee that the system is controllable?

Prob 7 | The linearized longitudinal motion of a helicopter near hover (figure below) can be modeled by the normalized third-order system

$$\begin{bmatrix} \dot{q} \\ \dot{\theta} \\ \dot{u} \end{bmatrix} = \begin{bmatrix} -0.4 & 0 & -0.01 \\ 1 & 0 & 0 \\ -1.4 & 9.8 & -0.02 \end{bmatrix} \begin{bmatrix} q \\ \theta \\ u \end{bmatrix} + \begin{bmatrix} 6.3 \\ 0 \\ 9.8 \end{bmatrix} \delta$$

where

$$\begin{aligned} q &= \text{pitch rate} \\ \theta &= \text{pitch angle of fuselage} \\ u &= \text{horizontal velocity (standard aircraft notation)} \\ \delta &= \text{rotor tilt angle (control variable)} \end{aligned}$$



Suppose our sensor measures the horizontal velocity u as the output, that is $y = u$
Use Matlab to answer the following questions.

a) Find the open-loop pole locations.

b) Is the system controllable?

c) Find the feedback gain that places the poles of the system at $s = -1 \pm 1j$ and $s = -2$