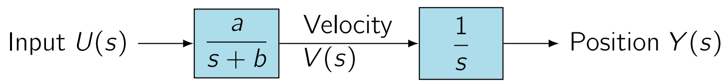
Problem 1.

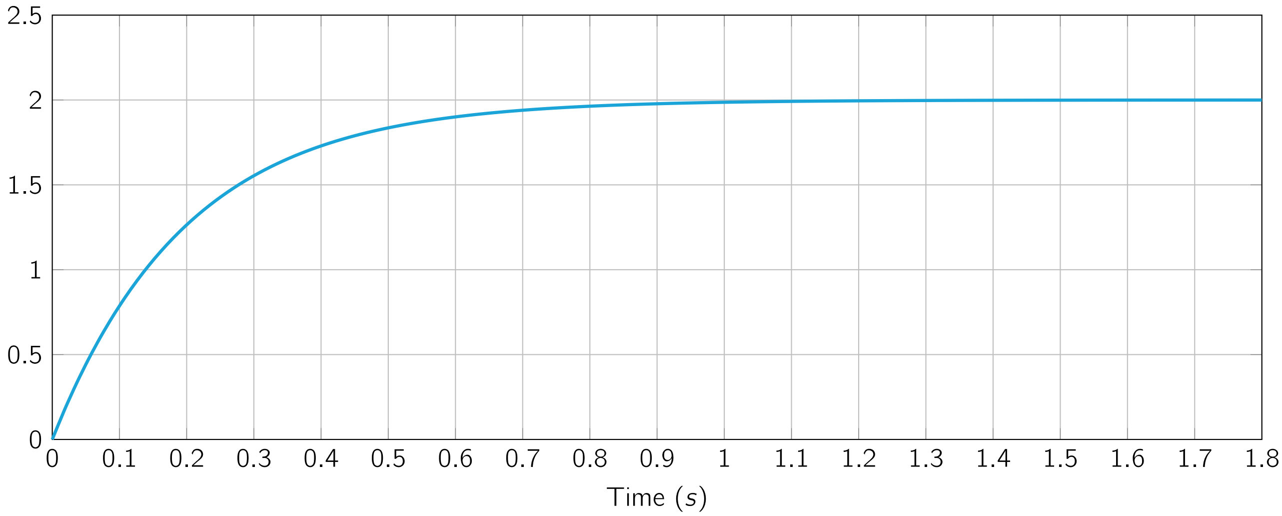
In this problem, you are to design a PD controller for the system below.



a) The figure below shows a response of the velocity $V(s)$ to a unit step input $U(s)$. Give constants a and b to best fit the model to this data.

$a =$ 10

$b =$ 5



b) The control specification is to ensure that the closed-loop poles lie within the shaded region shown in the figure below, where $\alpha = \sin^{-1}(0.96)$ and $\beta = \sin^{-1}(0.8)$.

Give the maximum and minimum for each of the properties below that the specification allows in response to a unit step input

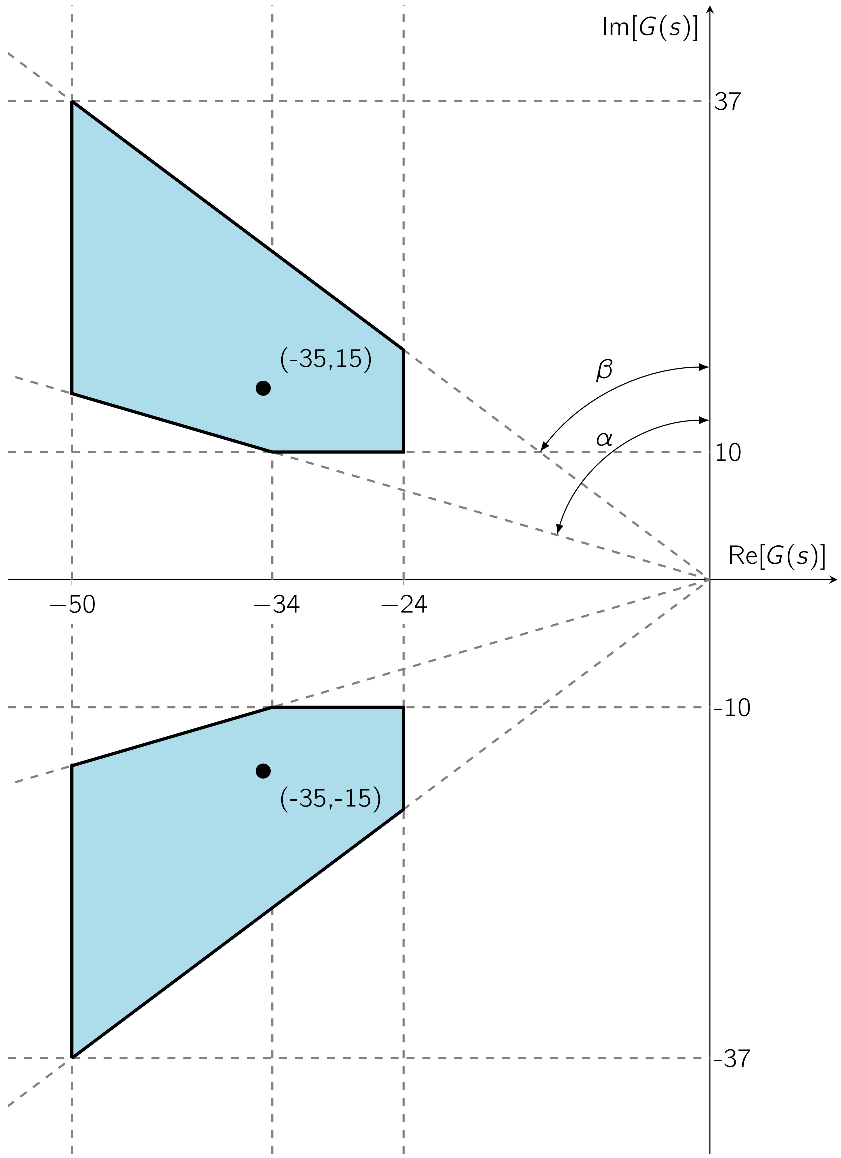
	Min	Max
Percent overshoot	0.002%	1.5%
Settling time (2%)	0.08	0.17
Peak time	0.085	0.31

Reading the plot, we see that

$$24 \le \sigma \le 50 \quad \rightarrow \quad \frac{4}{50} = 0.08 \le T_s \le \frac{4}{24} = 0.17$$

$$0.8 \le \zeta \le 0.96 \quad \rightarrow \quad 100e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.002\% \le PO \le 1.5\%$$

$$10 \le \omega_d \le 37 \quad \rightarrow \quad \frac{\pi}{\omega_d} = \frac{\pi}{37} = 0.085 \le T_p \le \frac{\pi}{10} = 0.31$$

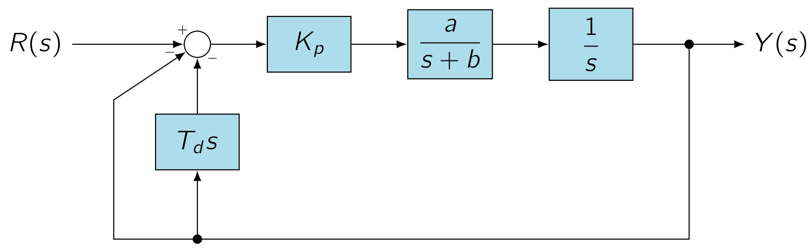


Im[$G(s)$]
37
(-35,15)
β
α
10
Re[$G(s)$]
-50
-34
-24
-10
(-35,-15)
-37

c) Design a PD controller such that the specification given in the previous part is met for the system. Choose your controller so that the closed-loop poles are as those marked on the figure $-35 \pm 15i$. The structure of the control system is shown below (recall that we don't take the derivative of the reference).

$K_p =$ 145

$T_d =$ 0.045



Compute the closed-loop transfer function

$$Y = \frac{1}{s}\frac{a}{s+b}K_p\left(R - Y - T_d sY\right)$$
$$\frac{Y}{R} = \frac{aK_p}{s^2 + s(b + aK_pT_d) + aK_p}$$
$$= \frac{10K_p}{s^2 + s(5 + 10K_pT_d) + 10K_p}$$

From the points given on the figure, we form the desired characteristic equation

$$(s - (-35 + 15i))(s - (-35 - 15i)) = s^2 - s(-35 - 15i - 35 + 15i) + 35^2 + 15^2$$
$$= s^2 + 70s + 1450$$

Matching the characteristic equations, we solve for K_p and T_d

$$10K_p = 1450 \quad \rightarrow \quad K_p = 145$$
$$5 + 10K_pT_d = 70 \quad \rightarrow \quad T_d = \frac{6.5}{145} = 0.045$$

Problem 2.

Consider the system

$$G(s) = \frac{s+1}{s^2+3s+4}$$

a) Give the system in control canonical form $\dot{x} = Ax + Bu$, $y = Cx + Du$

$$A = \begin{bmatrix} -3 & -4 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 \end{bmatrix} \qquad D = 0$$

b) Design a full state feedback controller that places the closed-loop poles at -3 and -4.

$$K = \begin{bmatrix} 4 & 8 \end{bmatrix}$$

$$\begin{aligned}
\det(sI - A + BK) &= \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & -4 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} K_1 & K_2 \end{bmatrix} \right| \\
&= \left| \begin{bmatrix} s+3+K_1 & 4+K_2 \\ -1 & s \end{bmatrix} \right| \\
&= s^2 + (3+K_1)s + 4 + K_2 \\
&= (s+3)(s+4) = s^2 + 7s + 12
\end{aligned}$$

which gives the solution $K = \begin{bmatrix} 4 & 8 \end{bmatrix}$

c) Design an estimator for the system that places the estimator poles $3\times$ faster than the controlled poles

$$L = \begin{bmatrix} 61 \\ -43 \end{bmatrix}$$

$$\begin{aligned}
\det(sI - A + LC) &= \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & -4 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} \right| \\
&= \left| \begin{bmatrix} s+3+L_1 & 4+L_1 \\ -1+L_2 & s+L_2 \end{bmatrix} \right| \\
&= s^2 + s(L_2+L_1+3) + L_2(3+L_1) - (4+L_1)(-1+L_2) \\
&= (s+9)(s+12) = s^2 + 21s + 108
\end{aligned}$$

Solving gives $L_1 = 61$ and $L_2 = -43$

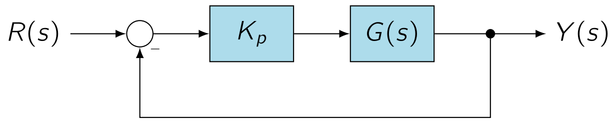
d) What are the poles of the resulting closed-loop system?

$$\begin{bmatrix} -3 & -4 & -9 & -12 \end{bmatrix}$$

Separation principle means that the closed-loop poles are those of the controlled system and the estimator.

Problem 3.

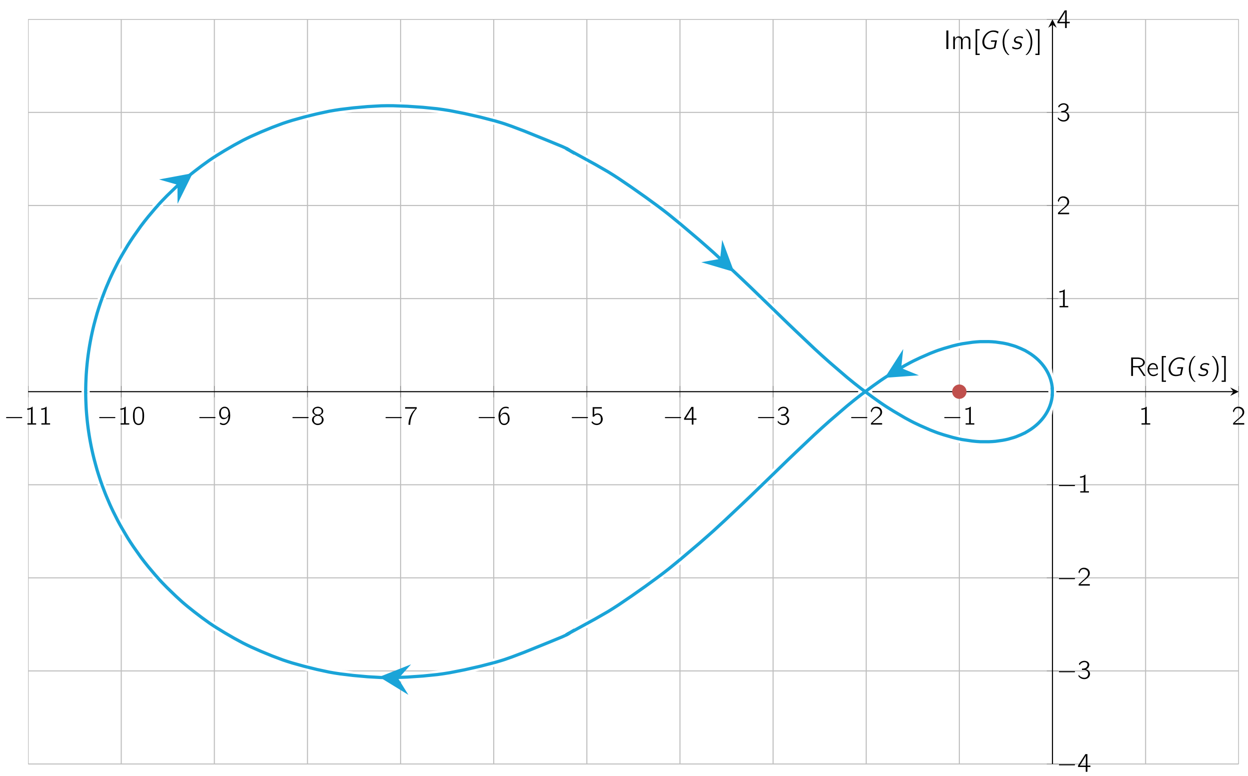
Consider the following closed-loop system with a proportional controller K_p



a) The Nyquist plot for the system $5.2\frac{(s+3)(s+2)}{(s-3)(s+1)^2}$ is shown below.

Give the range of values of K_p for which the closed-loop system will be stable

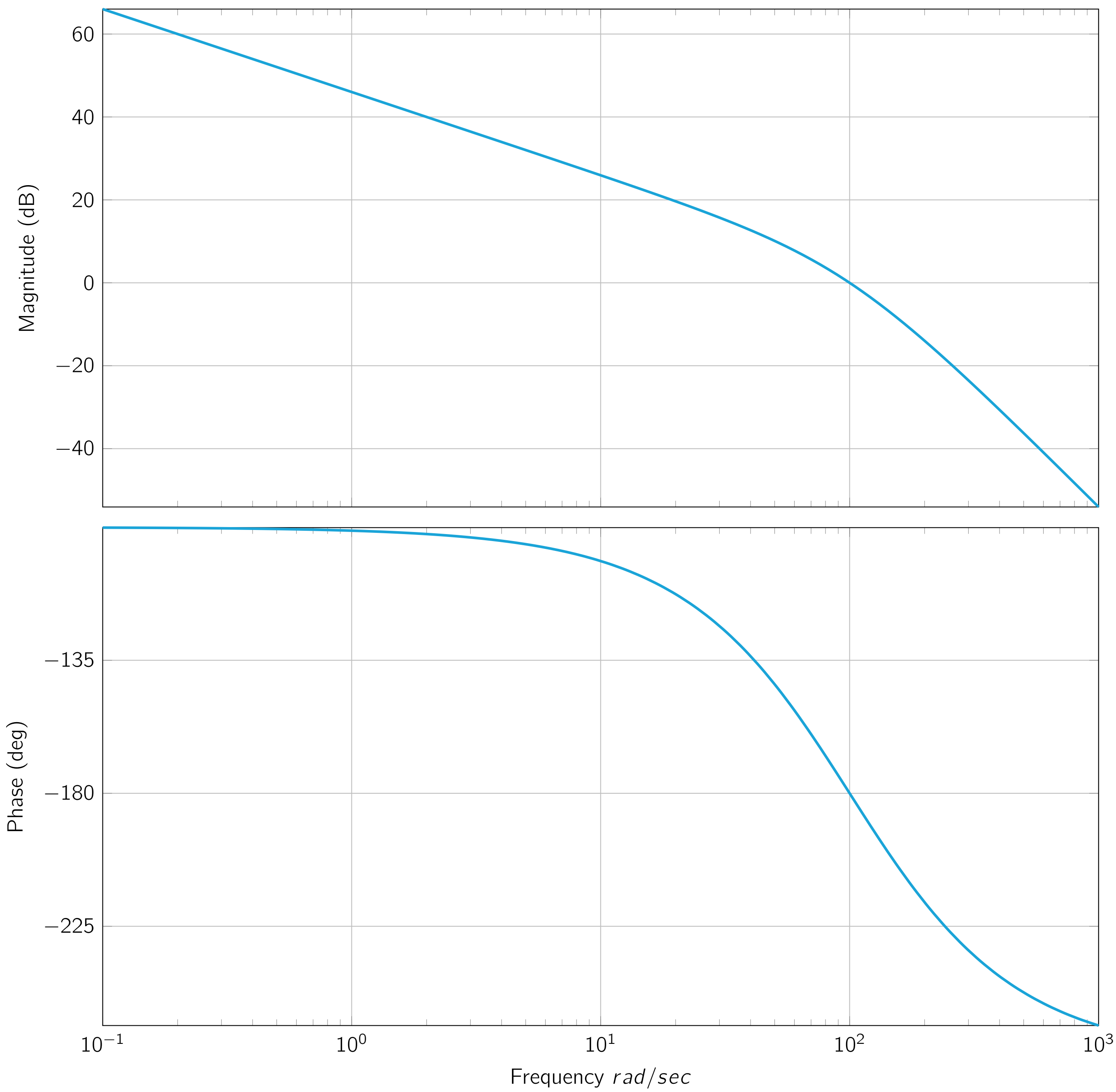
$K_p \geq 0.5$



b) Sketch the Bode plot of the system $G(s) = \dfrac{2,000,000}{s^3 + 200s^2 + 10,000s}$ and estimate the gain and phase margins.

Gain margin ≈ 0 (dB) Phase margin ≈ 0 (degrees)

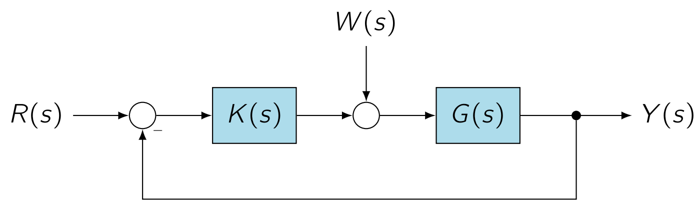
Will the closed-loop system be stable if $K_p = 5$? No (yes/no)

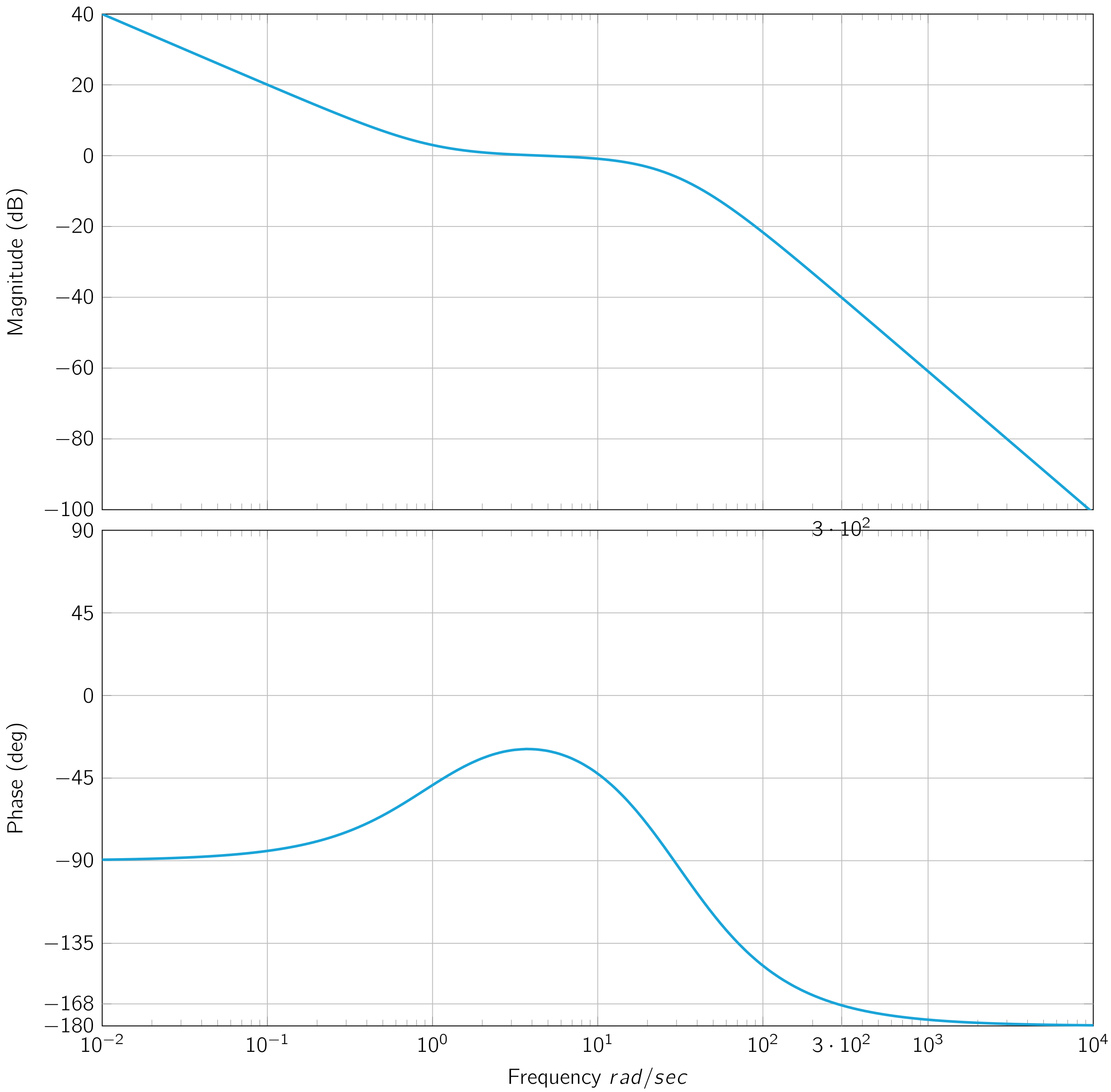


Problem 4.

Shown below is the Bode plot of the system

$$G(s) = \frac{s+1}{s(s/30+1)^2}$$





a) Design a lead compensator and proportional gain to satisfy the following criteria

- Crossover frequency of around 300r/s
- Phase margin of 50 degrees

$K(s) =$

b) What is the steady-state error of the closed-loop system with your controller in response to:

Unit step of the reference $R(s)$ Unit step of the disturbance $W(s)$

c) Add a lag compensator to your controller so that the following criteria are met

- Crossover frequency of around 300r/s
- Phase margin of at least 30 degrees
- Zero steady-state error in response to a unit step input in the disturbance
- Zero steady-state error in response to a unit ramp input in the reference

$K(s) =$

Problem 5.

Consider the dynamic system

$$\dot{x} = \begin{bmatrix} -5 & -4 \\ -4 & -5 \end{bmatrix} x + Bu$$
$$y = Cx$$

a) For which of the following matrices is the system controllable and/or observable?

	Controllable? (yes/no)	Observable? (yes/no)
$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \end{bmatrix}$	No	No
$B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \end{bmatrix}$	Yes	No
$B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 1 \end{bmatrix}$	Yes	Yes
$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 1 \end{bmatrix}$	No	Yes

For the remaining question parts, assume that $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and that the controller $u = -\begin{bmatrix} 1 & 2 \end{bmatrix} x + Nr$ is applied to the system, where r is a reference input and N is a constant.

b) Give the transfer function from the reference r to the output of the system y, parameterized by N.

$$\frac{Y(s)}{R(s)} = \boxed{N\frac{s+5}{s^2+11s+6}}$$

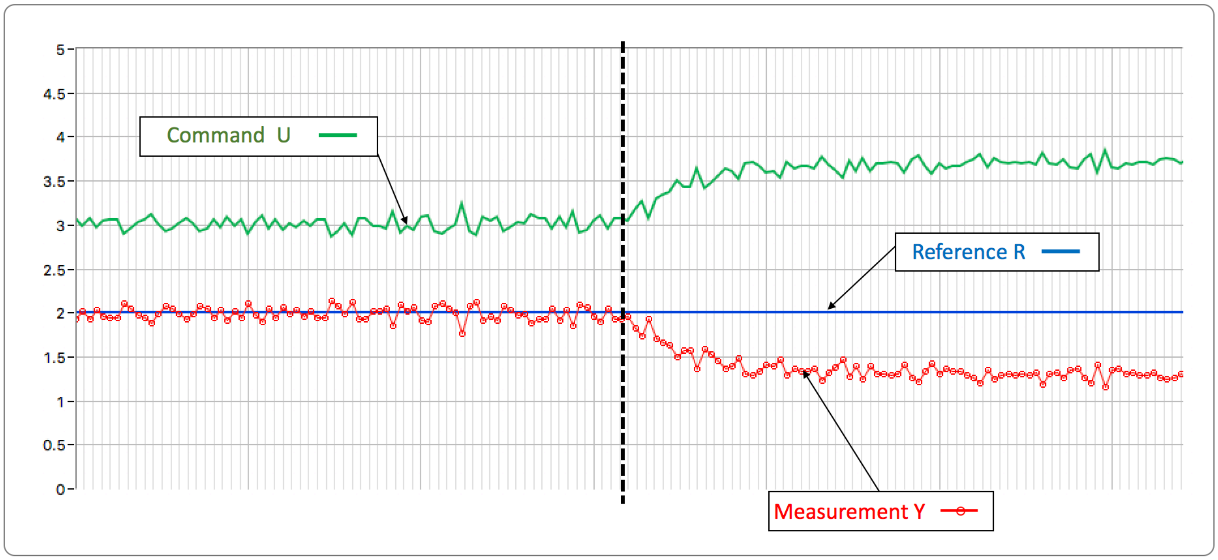
c) Give a value for N so that the output y will equal r in steady-state if r is a step input.

$$N = \boxed{\frac{6}{5}}$$

Problem 6. Travaux Pratiques

Notes:

- The answer and the justification must be correct to get the points.
- Measurements have been made on the electrical drives used during the labs.

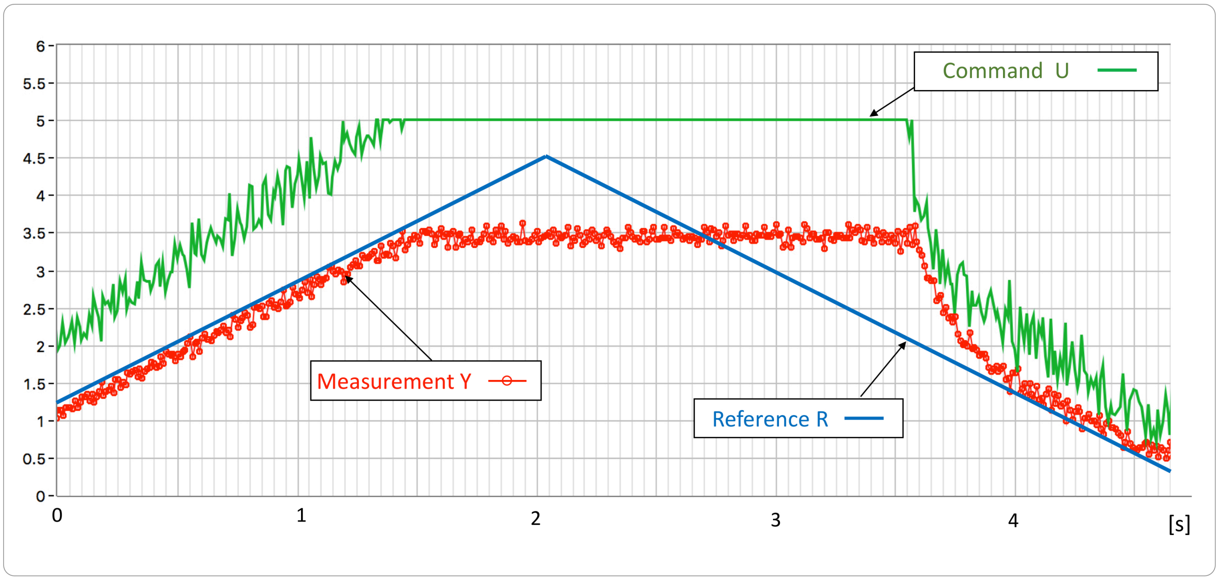


1a) A constant perturbation has been applied to the system at the dashed line. What type of control structure is implemented? Justify.

Speed control → PI or P + U0, but since it cannot compensate for the perturbation → P + U0

1b) Compute the controller parameters.

U0 = 3v since U = (R-Y)*Kp+U0, 3 = (2-2)*Kp + U0, measured before the perturbation
Kp = 1, since U = (R-Y)*Kp+U0, 3.75 = (2-1.25)*Kp + 3, measured after the perturbation



2a) What is the controller structure? Justify.

PI since speed ctrl and permanent err

2b) Between 1.5 [s] and 3.5 [s] the command remains saturated, why? Justify.

The command was saturated between 1.5 and 2.8s, cannot reach the desired value Between 2.8-3.5s still saturated since Ui was increasing internally and needs to be emptied before the output can follow the reference again (-¿ ARW not actif)