Exercise 1

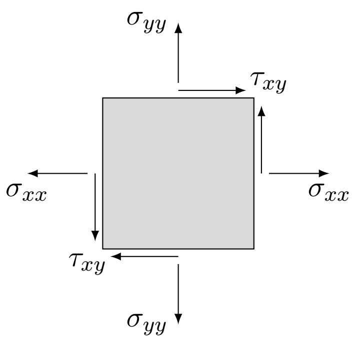


Figure 1: The state of stress on an element of a structure

A 2D stress element is subjected to the normal stresses $\sigma_{xx} = 50\,\text{MPa}$, $\sigma_{yy} = 10\,\text{MPa}$ and shear stress $\tau_{xy} = -20\,\text{MPa}$ shown in the drawing in figure 1. We know that the material of the element has a weak axis rotated 30° counter clockwise.

a) What are the normal and shear stresses along that axis? Calculate once with the formulas we derived in class and once with matrix rotation.

As discussed in class, there exists for every stress state a set of directions in which the normal stresses are maximum and minimum, and the shear stresses are equal to zero. These axes are called the principal axes, and the corresponding stresses are called the principal stresses.

b) For the stress state above, calculate the principal stresses and the principal axes using the formulas from the formula sheet.

c) Calculate the principal stresses and the principal axes of the element using the stress tensor. *Hint: the principal stresses are the eigenvalues of the stress tensor, while the principal axes are given by the eigenvectors.*

Exercise solution 1

Given:

- Normal stresses: $\sigma_{xx} = 50\,\text{MPa}$, $\sigma_{yy} = 10\,\text{MPa}$.
- Shear stress: $\tau_{xy} = -20\,\text{MPa}$

Asked:

- Stresses when rotated 30° CCW by formulas and by matrix transformation.
- The principal stresses.
- The directions of the principal stresses.

Relevant relationships: Stress transformations:

$$\begin{aligned}
\sigma_{x'} &= \frac{\sigma_{xx}+\sigma_{yy}}{2} + \frac{\sigma_{xx}-\sigma_{yy}}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \\
\sigma_{y'} &= \frac{\sigma_{xx}+\sigma_{yy}}{2} - \frac{\sigma_{xx}-\sigma_{yy}}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \\
\tau_{x'y'} &= -\frac{\sigma_{xx}-\sigma_{yy}}{2}\sin 2\theta + \tau_{xy}\cos 2\theta
\end{aligned}$$

a)

With formulas: We calculate the double angle sines (CCW is positive!) to get:

$$\sin(2\cdot 30°) = \frac{\sqrt{3}}{2} \qquad \cos(2\cdot 30°) = \frac{1}{2}$$

And with that we can find the rotated stress states as:

$$\begin{aligned}
\sigma'_{xx} &= \frac{50\,\text{MPa}+10\,\text{MPa}}{2} + \frac{50\,\text{MPa}-10\,\text{MPa}}{2}\cdot\frac{1}{2} + -20\,\text{MPa}\cdot\frac{\sqrt{3}}{2} \\
&= \left(40-10\sqrt{3}\right) = 22.6\text{MPa} \\
\sigma'_{yy} &= \frac{50\,\text{MPa}+10\,\text{MPa}}{2} - \frac{50\,\text{MPa}-10\,\text{MPa}}{2}\cdot\frac{1}{2} - -20\,\text{MPa}\cdot\frac{\sqrt{3}}{2} \\
&= \left(20+10\sqrt{3}\right) = 37.3\text{MPa} \\
\tau'_{xy} &= \frac{-50\,\text{MPa}-10\,\text{MPa}}{2}\cdot\frac{\sqrt{3}}{2} + -20\,\text{MPa}\cdot\frac{1}{2} = \left(-10-10\sqrt{3}\right) = -27.3\text{MPa}
\end{aligned}$$

By matrix rotation: We first get the rotation matrix and its inverse:

$$\boldsymbol{R} = \begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix} = \begin{bmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix} \qquad \boldsymbol{R}^{-1} = \boldsymbol{R}^{\mathsf{T}} = \begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$$

at which point we can find the rotated stress tensor from the original one:

$$\begin{aligned}
\boldsymbol{\sigma}' = \boldsymbol{R}^{-1}\cdot\boldsymbol{\sigma}\cdot\boldsymbol{R} &= \begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}\begin{bmatrix}50 & -20 \\ -20 & 10\end{bmatrix}\text{MPa}\begin{bmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix} \\
&= \begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}\begin{bmatrix}25\sqrt{3}-10 & -25-10\sqrt{3} \\ 5-10\sqrt{3} & 10+5\sqrt{3}\end{bmatrix}\text{MPa} \\
&= \begin{bmatrix}\frac{1}{2}\left(25\sqrt{3}-10\right)\sqrt{3}+\frac{5}{2}-5\sqrt{3} & -\frac{25}{2}\sqrt{3}+5+\frac{1}{2}\left(5-10\sqrt{3}\right)\sqrt{3} \\ \frac{1}{2}\left(-25-10\sqrt{3}\right)\sqrt{3}+5+\frac{5}{2}\sqrt{3} & \frac{25}{2}+5\sqrt{3}+\frac{1}{2}\left(10+5\sqrt{3}\right)\sqrt{3}\end{bmatrix}\text{MPa} \\
&= \begin{bmatrix}40-10\sqrt{3} & -10-10\sqrt{3} \\ -10-10\sqrt{3} & 20+10\sqrt{3}\end{bmatrix}\text{MPa} \\
&= \begin{bmatrix}22.6 & -27.3 \\ -27.3 & 37.3\end{bmatrix}\text{MPa}
\end{aligned}$$

b)

According to the formula sheet, the principal stresses in 2D are:

$$\begin{aligned}\sigma_1 &= \frac{\sigma_{xx}+\sigma_{yy}}{2} + \sqrt{(\frac{\sigma_{xx}-\sigma_{yy}}{2})^2 + \tau_{xy}^2} \\ &= \frac{50+10}{2} + \sqrt{(\frac{50-10}{2})^2 + 20^2} \\ &= 58.3\text{MPa}\end{aligned}$$

$$\begin{aligned}\sigma_2 &= \frac{\sigma_{xx}+\sigma_{yy}}{2} - \sqrt{(\frac{\sigma_{xx}-\sigma_{yy}}{2})^2 + \tau_{xy}^2} \\ &= \frac{50+10}{2} - \sqrt{(\frac{50-10}{2})^2 + 20^2} \\ &= 1.7\text{MPa}\end{aligned}$$

The principal axes are given by:

$$\begin{aligned}\tan 2\theta_p &= \frac{2\tau_{xy}}{\sigma_{xx}-\sigma_{yy}} \\ \theta_p &= \frac{1}{2}\cdot \arctan\frac{2\tau_{xy}}{\sigma_{xx}-\sigma_{yy}} = \frac{1}{2}\cdot\arctan\frac{2\cdot(-20)}{50-10} \\ \theta_{p,1} &= -22.5° \\ \theta_{p,2} &= 67.5°\end{aligned}$$

Note that both axes are orthogonal.

c)

Principal stresses: By definition, the principal stresses are the eigenvalues of the stress tensor.

$$\begin{aligned}det\left(\begin{bmatrix}50 & -20\\ -20 & 10\end{bmatrix} - \sigma\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right) &= 0 \\ det\left(\begin{bmatrix}50-\sigma & -20\\ -20 & 10-\sigma\end{bmatrix}\right) &= 0 \\ (50-\sigma)(10-\sigma)-(-20)(-20) &= 0 \\ \sigma^2 - 60\sigma + 100 &= 0\end{aligned}$$

The solutions of the equation $\sigma^2 - 60\sigma + 100 = 0$ are:

$$\sigma_1 = 30 + 20\sqrt{2} = 58.3\text{MPa} \qquad \sigma_2 = 30 - 20\sqrt{2} = 1.7\text{MPa}$$

Principal axes: The principal axes are the eigenvectors of the stress tensor.

The first principal axis e_1 corresponds to the main stress σ_1. It is defined by a vector solution of the equation $\boldsymbol{\sigma} \cdot e_1 = \sigma_1 \cdot e_1$.

$$\begin{bmatrix} 50 & -20 \\ -20 & 10 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} - (30 + 20\sqrt{2}) \begin{bmatrix} x \\ y \end{bmatrix} = 0$$
$$20 \begin{bmatrix} (1-\sqrt{2})x - y \\ -x - (1+\sqrt{2})y \end{bmatrix} = 0$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}-1} \\ 1 \end{bmatrix}$$

And the first principal axis is defined by the vector:

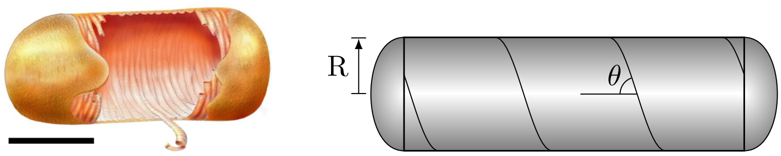
$$e_1 = \begin{bmatrix} -\frac{1}{\sqrt{2}-1} \\ 1 \end{bmatrix} \quad \rightarrow \quad \theta_{p,1} = \arctan -(\sqrt{2}-1) = -22.5°$$

In the same manner, e_2 is solution of $\boldsymbol{\sigma} \cdot e_2 = \sigma_2 \cdot e_2$:

$$\begin{bmatrix} 50 & -20 \\ -20 & 10 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} - (30 - 20\sqrt{2}) \begin{bmatrix} x \\ y \end{bmatrix} = 0$$
$$20 \begin{bmatrix} (1+\sqrt{2})x - y \\ -x - (1-\sqrt{2})y \end{bmatrix} = 0$$
$$e_2 = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}+1} \\ 1 \end{bmatrix} \quad \rightarrow \quad \theta_{p,2} = \arctan(\sqrt{2}+1) = 67.5°$$

One can notice that $e_1 \cdot e_2 = -\frac{1}{\sqrt{2}-1} \cdot \frac{1}{\sqrt{2}+1} + 1 \cdot 1 = 0$ which shows again that the two principal axes are orthogonal. This is not surprising as the eigenvectors of a symmetric matrix (and $\boldsymbol{\sigma}$ is a symmetric matrix) are always orthogonal according to the spectral theorem.

Exercise 2



(a) Bacterial cell wall architecture. Scale bar is 1 µm. (b) Very simplified model of a bacterial peptidoglycan structure.

Figure 2: Gram-negative bacterium.

In a gram-negative bacterium, the peptidoglycan layer is situated between the inner and outer lipid bi-layer. This peptidoglycan layer is believed to be responsible for the mechanical stability and robustness of the bacterial wall.

Figure 2a depicts the architecture of the peptidoglycan, which a recent study has revealed[1].
We simplify the shown peptidoglycan architecture and consider the model as shown in figure 2b. The bacterium has an internal pressure p of 150 kPa. The peptidoglycan cables form an angle $\theta = 55°$ with the longitudinal axis of the cylinder. Consider only the peptidoglycan layer as the bacterial wall with a thickness of $t = 50$ nm, a Young's modulus $E = 10$ MPa and a Poisson ratio of $\nu = 0.3$. The rod shaped bacterium has a radius of $R = 0.5$ µm. Calculate:

a) The principal stresses in an element in the cylindrical part of the bacterium. To do so, consider a 2D element of the bacterial wall, assuming plane stress.

b) The tensile and shear stresses in an element aligned to the peptidoglycan cables.

c) The longitudinal and hoop strains in the bacterium.

Hint: Use the approximations for stress in thin walled structures.

Exercise solution 2

Given:

- Geomerty: Radius $R = 0.5$ µm, wall thickness $t = 50$ nm, angle of peptidoglycan cable alignment $\theta = 55°$.
- Inside pressure p of 150 kPa.
- Material properties $E = 10$ MPa, $\nu = 0.3$.

Asked:

- Principal stresses σ_1 and σ_2.
- Shear and normal stresses along the bacterium.
- Hoop and longitudinal strains in the bacterium.

Relevant relationships:

- Hoop stress σ_H and longitudinal stress σ_L with pressure p , thickness t and radius R.

$$\sigma_H = \frac{pR}{t} \qquad \sigma_L = \frac{pR}{2t}$$

[1] Hayhurst, Emma J., et al. "Cell wall peptidoglycan architecture in Bacillus subtilis." Proceedings of the National Academy of Sciences 105.38 (2008): 14603-14608.

- Transformation equations for plane stress :

$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\sigma'_y = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) - \tau_{xy}\sin(2\theta)$$

$$\tau'_{xy} = -\frac{\sigma_x - \sigma_y}{2}\sin(2\theta) + \tau_{xy}\cos(2\theta)$$

- Strains in the plane

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right) \quad \varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x\right)$$

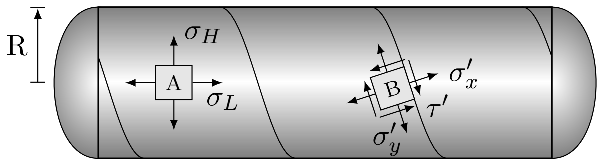


Figure 3: Simplified bacterial wall model with added elements.

a)

We get the principal stresses along the hoop and longitudinal dimensions, since no additional shear loads act on the structure. Thus:

$$\sigma_1 = \sigma_H = \frac{pR}{t} = \frac{150\,\text{kPa}\cdot 0.5\,\mu\text{m}}{50\,\text{nm}} = 1.5\,\text{MPa} \qquad \sigma_2 = \sigma_L = \frac{pR}{2t} = \frac{\sigma_1}{2} = 750\,\text{kPa}$$

b)

Knowing that $\sigma_x = \sigma_2 = 750\,\text{kPa}$, $\sigma_y = \sigma_1 = 1.5\,\text{MPa}$ and $\tau_{xy} = 0\,\text{MPa}$, we can apply the transformation equations for plane stress.
Using the formulas for rotation around the angle $\theta' = -\theta = -55°$ of the stress we get

$$\sigma'_x = \frac{\sigma_L + \sigma_H}{2} + \frac{\sigma_L - \sigma_H}{2}\cos(2\theta') = 1.253\,\text{MPa}$$
$$\sigma'_y = \frac{\sigma_L + \sigma_H}{2} - \frac{\sigma_L - \sigma_H}{2}\cos(2\theta') = 997\,\text{kPa}$$
$$\tau_{x'y'} = -\frac{\sigma_L - \sigma_H}{2}\sin(2\theta') = -352\,\text{kPa}$$

c)

Longitudinal strain:

$$\varepsilon_2 = \varepsilon_L = \frac{1}{E}\left(\sigma_L - \nu\sigma_H\right) = 30 \times 10^{-3}$$

Hoop strain:

$$\varepsilon_1 = \varepsilon_H = \frac{1}{E}\left(\sigma_H - \nu\sigma_L\right) = 128 \times 10^{-3}$$

Exercise 3

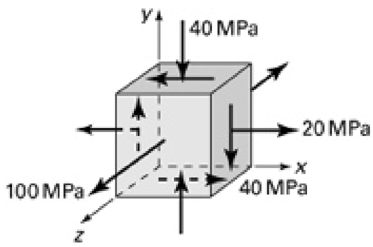


Figure 4: The state of stress on an element of a structure

The state of stress on an element of a structure is illustrated in Figure 4.

a) Determine the principal stresses σ_1, σ_2 and σ_3.

b) Deduce from it the maximum shear stress $\tau_{\max}$, knowing that

$$\tau_{\max} = \frac{\max(\sigma_1, \sigma_2, \sigma_3) - \min(\sigma_1, \sigma_2, \sigma_3)}{2}$$

Exercise solution 3

Given:

- normal stresses: $\sigma_x = 20\,\text{MPa}$, $\sigma_y = -40\,\text{MPa}$ and $\sigma_z = 100\,\text{MPa}$
- shear stresses: $\tau_{xy} = -40\,\text{MPa}$, $\tau_{yz} = 0\,\text{MPa}$ and $\tau_{xz} = 0\,\text{MPa}$
- the maximum shear stress $\tau_{max} = \frac{max(\sigma_1,\sigma_2,\sigma_3) - min(\sigma_1,\sigma_2,\sigma_3)}{2}$

Asked:

- the principal stresses
- the maximum shear stress

Relevant relationships:

- the characteristic equation: $\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$ with

$$I_1 = \sigma_x + \sigma_y + \sigma_z$$

$$I_2 = \sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{xz}^2 - \tau_{yz}^2$$

$$I_3 = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{xz}\tau_{yz} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{xz}^2 - \sigma_z\tau_{xy}^2$$

a)

The stress state can be written as the stress tensor:

$$\overleftrightarrow{\sigma} = \begin{bmatrix} 20 & -40 & 0 \\ -40 & -40 & 0 \\ 0 & 0 & 100 \end{bmatrix} \text{MPa}$$

Since shear stresses τ_{xz} and τ_{yz} equal 0 then we can see that the $\sigma_z = 100\,\text{MPa}$ is already one principal stress. One way we can see this is because principal stresses are the solutions of the equation:

$$\det\left(\overleftrightarrow{\sigma} - \lambda\overleftrightarrow{E}\right) = 0$$

So once we solve this equation and find all λ that satisfy the equation, these will be all the principal stresses (these are the roots of our characteristic equation $\lambda^3 - I_1\lambda^2 + I_2\lambda - I3 = 0$, only we wrote σ instead of λ on lectures). For the case of our stress tensor matrix we can see that the mentioned equation can be written as

$$\det\left(\begin{bmatrix} 20-\lambda & -40 & 0 \\ -40 & -40-\lambda & 0 \\ 0 & 0 & 100-\lambda \end{bmatrix}\right) =$$

$$(100\,\text{MPa} - \lambda) \cdot \det\left(\begin{bmatrix} 20-\lambda & -40 \\ -40 & -40-\lambda \end{bmatrix}\right) = 0$$

$$(100\,\text{MPa} - \lambda) \cdot \det\left(\begin{bmatrix} 20 & -40 \\ -40 & -40 \end{bmatrix} - \lambda\overleftrightarrow{E}\right) = 0$$

So we see that $\lambda = 100\,\text{MPa}$ really satisfies the equation.

To get other principal stresses, we calculate the stress invariants I_1, I_2 and I_3:

$$I_1 = \sigma_x + \sigma_y + \sigma_z = 80\,\text{MPa}$$

$$I_2 = \sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 = -4400\,\text{MPa}^2$$

$$I_3 = \sigma_x\sigma_y\sigma_z - \sigma_z\tau_{xy}^2 = -240\,000\,\text{MPa}^3$$

Further out I will omit writing units in the characteristic equation for the sake of easier writing.

We can now write the characteristic equation as

$$\sigma^3 - 80\sigma^2 - 4400\sigma + 240000 = 0$$

Since we know that one principal stress equals $\sigma_1 = 100\,\mathrm{MPa}$ we can divide the characteristic equation with $\sigma - 100$ to get other two principal stresses:

$$(\sigma - 100)(\sigma^2 + 20\sigma - 2400) = 0$$

Solving the second order polynomial on the right we get values of other two principal stresses:

$$\sigma_2 = 40\,\mathrm{MPa} \text{ and } \sigma_3 = -60\,\mathrm{MPa}$$

b)

The maximum shear force can be determined with the relation :

$$\tau_{max} = \frac{\max(\sigma_1, \sigma_2, \sigma_3) - \min(\sigma_1, \sigma_2, \sigma_3)}{2}$$

In our case, we get :

$$\tau_{max} = \frac{\sigma_3 - \sigma_1}{2} = 80\,\mathrm{MPa}$$

Exercise 4

A wire strain gauge can effectively measure strain in only one direction. To determine the three independent components of plane strain, three linearly independent strain measures are needed, i.e., three strain gauges positioned in a rosette-like layout.

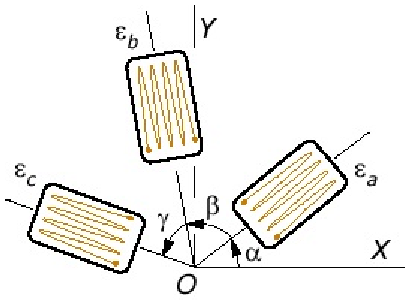


Figure 5: Rosette-like strain layout

a) Determine the value of ε_a, ε_b and ε_c as a function of ε_x, ε_y, γ_{xy}, α, β and γ. You can derive the transformation formulas for strain in analogy to the stress transformations.

b) For $\alpha = 30^o$ and $\beta = \gamma = 60^o$ (60^o strain rosette configuration), you measure the following strains : $\varepsilon_a = 300\mu m/m$, $\varepsilon_b = 100\mu m/m$ and $\varepsilon_c = 10\mu m/m$. Determine the values of ε_x, ε_y and $\gamma_x y$.

Exercise solution 4

Given:

- Rosette layout
- angles : $\alpha = 30^o$, $\beta = \gamma = 60^o$ and strain :$\varepsilon_a = 300\mu m/m$, $\varepsilon_b = 100\mu m/m$ and $\varepsilon_c = 10\mu m/m$

Asked:

- value of ε_a, ε_b and ε_c as a function of ε_x, ε_y, γ_{xy}, α, β and γ
- values of ε_x, ε_y and $\gamma_x y$ for given measured strains $\varepsilon_a = 300\mu m/m$, $\varepsilon_b = 100\mu m/m$ and $\varepsilon_c = 10\mu m/m$

Relevant relationships: $\varepsilon'_x(\theta) = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos(2\theta) + \frac{\gamma_{xy}}{2}\sin(2\theta)$

a)

The strain $\boldsymbol{\varepsilon(\theta)}$ in direction θ can be determined thanks to a matrix coordinate transform. In this exercise, we are interested only in the strain $\varepsilon_x(\theta) = \boldsymbol{\varepsilon(\theta)}_{1,1}$:

$$\boldsymbol{\varepsilon(\theta)} = \begin{bmatrix} cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}^{-1} \cdot \begin{bmatrix} \varepsilon_x & \frac{\gamma_{xy}}{2} \\ \frac{\gamma_{xy}}{2} & \varepsilon_y \end{bmatrix} \cdot \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$
$$\boldsymbol{\varepsilon(\theta)}_{1,1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta) + \frac{\gamma_{xy}}{2} sin(2\theta)$$

(Note : this result is in the formulary)
As a consequence,

$$\varepsilon_a = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\alpha) + \frac{\gamma_{xy}}{2}\sin(2\alpha)$$
$$\varepsilon_b = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2(\alpha+\beta)) + \frac{\gamma_{xy}}{2}\sin(2(\alpha+\beta))$$
$$\varepsilon_c = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2(\alpha+\beta+\gamma)) + \frac{\gamma_{xy}}{2}\sin(2(\alpha+\beta+\gamma))$$

b)

When $\alpha = 30^o$ and $\beta = \gamma = 60^o$, the previous three equations become :

$$\varepsilon_a = \frac{3}{4}\varepsilon_x + \frac{1}{4}\varepsilon_y + \frac{\sqrt{3}}{4}\gamma_{xy} = 300\,\mu\text{m/m}$$
$$\varepsilon_b = \varepsilon_y = 100\,\mu\text{m/m}$$
$$\varepsilon_c = \frac{3}{4}\varepsilon_x + \frac{1}{4}\varepsilon_y - \frac{\sqrt{3}}{4}\gamma_{xy} = 10\,\mu\text{m/m}$$

And we can now deduce the strains in x and y directions, as well as γ_{xy} :

$$\varepsilon_x = 173\,\mu\text{m/m}$$
$$\varepsilon_y = 100\,\mu\text{m/m}$$
$$\gamma_{xy} = 335\,\mu\text{m/m}$$