Exercise 1

We consider a loaded beam with a cross-section in T-shape (fig. 1). The beam is characterized by the following values: $E = 20\,\text{GPa}$, $\nu = 0.2$, $t_1 = 2\,\text{cm}$, $t_2 = 1\,\text{cm}$, $\omega_1 = 1\,\text{cm}$, $\omega_2 = 4\,\text{cm}$, $L = 21\,\text{cm}$ and $F = 100\,\text{N}$. We define $z = 0$ at the top of the beam.

a) Find the moment of area (I_y) at the centroid. *Hint: you may quite literally find this result in the exercises of a previous week.*

b) Determine the expression of $q(x)$.

c) Calculate the bending line $\omega(x)$ of the beam. *Hint: a good engineer should be both smart and lazy. You can save yourself a lot of time and troubles here: re-use results from the previous exercises and apply the superposition principle.*

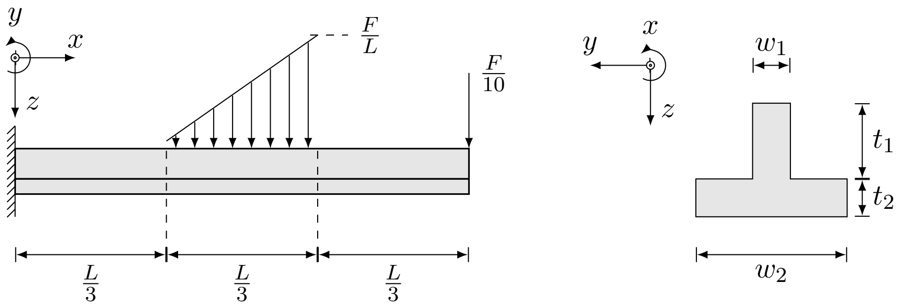

Figure 1: Schematic of the loaded bar and its cross-section.

Exercise solution 1

Given: Geometry, load.

Asked: Second moment of area I_y, $q(x)$ and $\omega(x)$.

Relevant relationships:

Steiner's Theorem

$$I_a = I_{cg} + Ad^2$$

Centroid Formula

$$z_c = \frac{\sum_i A_i z_{c,i}}{\sum_i A_i}$$

Deflection formula

$$\frac{d^2\omega_y(x)}{dx^2} = -\frac{1}{EI}M(x)$$

a)

Finding the centroid is straight-forward:

$$z_c = \frac{\sum_i A_i z_{c,i}}{\sum_i A_i} = \frac{w_1 t_1 z_{c,1} + w_2 t_2 z_{c,2}}{w_1 t_1 + w_2 t_2} = 2\ cm$$

As given in the formula section, $I_y = \int_A z^2 dA$. It is however easier to look at the cross-section as an assembly of two rectangles and use Steiner's Theorem.

$$\begin{aligned}
I_{y,1} &= \frac{w_1(t_1)^3}{12} = \frac{2}{3}\ cm^4 \\
I_{y,2} &= \frac{w_2(t_2)^3}{12} = \frac{1}{3}\ cm^4 \\
I_y^{z=z_c} &= I_{y,1} + w_1 t_1 (z_c - z_{c,1})^2 + I_{y,2} + w_2 t_2 (z_c - z_{c,2})^2 \\
&= \frac{2}{3} + 1 \cdot 2 \cdot (2-1)^2 + \frac{1}{3} + 4 \cdot 1 \cdot (2 - 2.5)^2 = 4\ cm^4
\end{aligned}$$

b)

The load function here is composed of a linearly distributed load that is 0 at $x = \frac{1}{3}L$ and increases linearly to $\frac{F}{L}$ at $x = \frac{2}{3}L$. Additionally, there is a point load at the end of the cantilever that we will ignore for now. It's contribution will be added later through superposition.

We write the load function as

$$q(x) = \frac{3F}{L^2}\left\langle x - \tfrac{L}{3}\right\rangle^1 - \frac{3F}{L^2}\left\langle x - \tfrac{2L}{3}\right\rangle^1 - \frac{F}{L}\left\langle x - \tfrac{2L}{3}\right\rangle^0$$

Note that even if we considered the point force, we would not need to add it in the load equation as it would be a boundary condition.

c)

Contribution of distributed load Integration and boundary conditions yield:

$$
\begin{aligned}
V(x) &= -\int q(x)\,\mathrm{d}x + C_1 \\
&= -\frac{3F}{2L^2}\left\langle x-\tfrac{L}{3}\right\rangle^2 + \frac{3F}{2L^2}\left\langle x-\tfrac{2L}{3}\right\rangle^2 + \frac{F}{L}\left\langle x-\tfrac{2L}{3}\right\rangle^1 + \frac{F}{6} \\
M(x) &= \int V(x)\,\mathrm{d}x + C_2 \\
&= -\frac{3F}{6L^2}\left\langle x-\tfrac{L}{3}\right\rangle^3 + \frac{3F}{6L^2}\left\langle x-\tfrac{2L}{3}\right\rangle^3 + \frac{F}{2L}\left\langle x-\tfrac{2L}{3}\right\rangle^2 + \frac{F}{6}x - \frac{5FL}{54}
\end{aligned}
$$

This was a result in a previous exercise session. From there, we integrate twice more and apply the boundary conditions of $\theta(0) = 0$ and $\omega(0) = 0$.

$$
\begin{aligned}
\theta(x) &= \frac{1}{EI}\Big(\int M(x)\,\mathrm{d}x + C_3\Big) = \frac{1}{EI}\Big[-\frac{3F}{24L^2}\left\langle x-\tfrac{L}{3}\right\rangle^4 + \frac{3F}{24L^2}\left\langle x-\tfrac{2L}{3}\right\rangle^4 \\
&\quad + \frac{F}{6L}\left\langle x-\tfrac{2L}{3}\right\rangle^3 + \frac{F}{12}x^2 - \frac{5FL}{54}x\Big] \\
\omega(x) &= -\int \theta(x)\,\mathrm{d}x + C_4 = \frac{1}{EI}\Big[\frac{F}{40L^2}\left\langle x-\tfrac{L}{3}\right\rangle^5 - \frac{F}{40L^2}\left\langle x-\tfrac{2L}{3}\right\rangle^5 \\
&\quad - \frac{F}{24L}\left\langle x-\tfrac{2L}{3}\right\rangle^4 - \frac{F}{36}x^3 + \frac{5FL}{108}x^2\Big]
\end{aligned}
$$

Contribution of point load A look into the formulary yields:

$$
\omega_2(x) = \frac{FL^3}{60EI}\Big[3\Big(\frac{x}{L}\Big)^2 - \Big(\frac{x}{L}\Big)^3\Big]
$$

Superposition Now all we have to do is add up the two contributions:

$$
\begin{aligned}
\omega_{tot}(x) &= \omega_2(x) + \omega(x) = \frac{FL^3}{60EI}\Big[3\Big(\frac{x}{L}\Big)^2 - \Big(\frac{x}{L}\Big)^3\Big] \\
&\quad + \frac{1}{EI}\Big[\frac{F}{40L^2}\left\langle x-\tfrac{L}{3}\right\rangle^5 - \frac{F}{40L^2}\left\langle x-\tfrac{2L}{3}\right\rangle^5 - \frac{F}{24L}\left\langle x-\tfrac{2L}{3}\right\rangle^4 - \frac{F}{36}x^3 + \frac{5FL}{108}x^2\Big] \\
&= \frac{F}{EI}\Big[\frac{1}{40L^2}\Big(\left\langle x-\tfrac{L}{3}\right\rangle^5 - \left\langle x-\tfrac{2L}{3}\right\rangle^5\Big) - \frac{1}{24L}\left\langle x-\tfrac{2L}{3}\right\rangle^4 - \frac{2}{45}x^3 + \frac{13L}{135}x^2\Big]
\end{aligned}
$$

The total deflection of the beam, as well as the contributions of the two separate loads is shown in fig. 2).

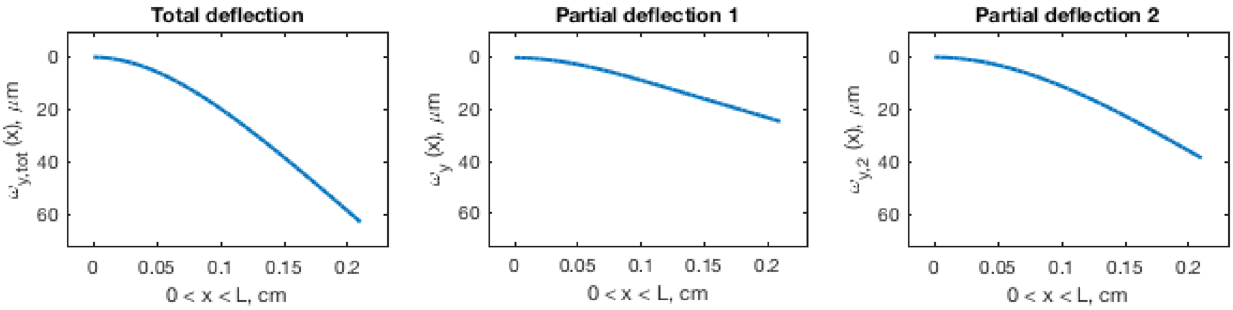

Figure 2: Beam deflection graphs

Exercise 2

The two beams in figure 3 are identically loaded with two forces and bend under that load. The beams have the same bending stiffness EI. Calculate the bending line of the beams with the help of singularity functions. You can neglect the effect of any axial forces (forces in x-direction).

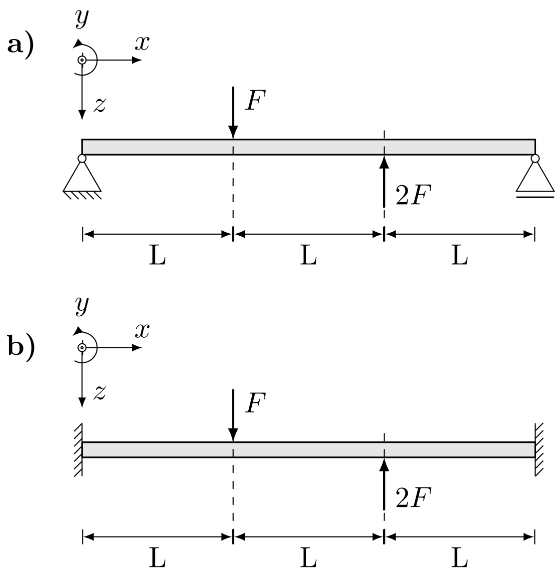

Figure 3: Double-supported beam under load.

Exercise solution 2

Given: Geometry, Force F, Bending stiffness EI.

Asked: Bending line of the loaded beam.

Relevant relationships: Bending equation

$$EI \cdot w^{(4)}(x) = EI \cdot \frac{\partial^4 w(x)}{\partial x^4} = q(x)$$

with distributed load $q(x)$.

Since the two beams have the same distributed load function, we only have to calculate the beam equation once. We will then find find a different set of integration constants for each case.

We have point loads at $x = L$ and $x = 2L$. So we get a distributed load of:

$$q(x) = F \cdot \langle x - L \rangle^{-1} - 2F \cdot \langle x - 2L \rangle^{-1}$$

Putting the bending equation and the distributed load together we get:

$$EI \cdot w^{(4)}(x) = F \cdot \langle x - L \rangle^{-1} - 2F \cdot \langle x - 2L \rangle^{-1}$$

By simple integration we get

$$\begin{aligned}
EI \cdot w^{(4)}(x) &= F \cdot \langle x - L \rangle^{-1} - 2F \cdot \langle x - 2L \rangle^{-1} = q(x) && (1)\\
EI \cdot w^{(3)}(x) &= F \cdot \langle x - L \rangle^{0} - 2F \cdot \langle x - 2L \rangle^{0} + C_1 = -V(x) && (2)\\
EI \cdot w^{(2)}(x) &= F \cdot \langle x - L \rangle^{1} - 2F \cdot \langle x - 2L \rangle^{1} + C_1 x + C_2 = -M(x) && (3)\\
EI \cdot w^{(1)}(x) &= \tfrac{1}{2} F \cdot \langle x - L \rangle^{2} - F \cdot \langle x - 2L \rangle^{2} + \tfrac{1}{2} C_1 x^2 + C_2 x + C_3 && (4)\\
&= -EI \cdot \theta(x) && (5)\\
EI \cdot w(x) &= \tfrac{1}{6} F \cdot \langle x - L \rangle^{3} - \tfrac{1}{3} F \cdot \langle x - 2L \rangle^{3} && (6)\\
&\quad + \tfrac{1}{6} C_1 x^3 + \tfrac{1}{2} C_2 x^2 + C_3 x + C_4 && (7)
\end{aligned}$$

a)

For finding boundary conditions we consider the physical system. We don't know any angles in the system. We also don't have endpoints where no forces act (both ends clamped). However the two rotating angles at the start and end of the beam will not extort any moments on the beam, thus we get:

$$M(0) = 0 \qquad M(3L) = 0$$

However, they will restrict the vertical deflection of the beam in both points, so we get:

$$w(0) = 0 \qquad w(3L) = 0$$

which are the four conditions necessary to fully define a fourth order differential equation such as the bending equation.

Using the boundary conditions we can determine values for C_{1-4}. By using $M(x = 0) = 0$ in (3) we get:

$$F \cdot \langle -L \rangle^1 - 2F \cdot \langle -2L \rangle^1 + C_1 \cdot 0 + C_2 = C_2 = 0$$

and by using $M(x = 3L) = 0$ again in the simplified equation (3):

$$F \cdot \langle 3L - L\rangle^1 - 2F \cdot \langle 3L - 2L\rangle^1 + C_1 \cdot 3L = F \cdot 2L - 2F \cdot L + C_1 \cdot 3L = C_1 = 0$$

So we already know $C_1 = 0$ and $C_2 = 0$. Using $w(x = 0) = 0$ in (7) we get:

$$EI \cdot 0 = \tfrac{1}{6}F \cdot \langle -L\rangle^3 - \tfrac{1}{3}F \cdot \langle -2L\rangle^3 + C_3 \cdot 0 + C_4 = C_4 = 0$$

Finally by applying $w(x = 3L) = 0$ in (7), we find:

$$\begin{aligned} EI \cdot 0 &= \tfrac{1}{6}F \cdot \langle 2L\rangle^3 - \tfrac{1}{3}F \cdot \langle L\rangle^3 + C_3 \cdot 3L \\ &= \tfrac{1}{6}F \cdot (2L)^3 - \tfrac{1}{3}F \cdot (L)^3 + C_3 \cdot 3L = -FL^3 + 3LC_3 = 0 \\ \rightarrow \quad C_3 &= -\tfrac{1}{3}FL^2 \end{aligned}$$

We can write the complete bending line formula as:

$$w(x) = \frac{1}{EI}\left[\tfrac{1}{6}F \cdot \langle x - L\rangle^3 - \tfrac{1}{3}F \cdot \langle x - 2L\rangle^3 - \tfrac{1}{3}FL^2x\right]$$

where the fact that all units within the bracket are $\mathrm{N\,m^3}$ with x[m], L[m] and F[N] is an easy check for integration mistakes.

b)

For the case where both sides are clamped, the system is overconstrained. However, we can use the same method here too.

The boundary conditions in this case are $w(0) = w(3L) = \theta(0) = \theta(3L) = 0$, since both walls can support a moment and a reactive shear force.
Using the functions (1)–(7) we first use $\theta(0) = 0$ to get $C_3 = 0$ and $w(0) = 0$ to get $C_4 = 0$.

The condition $\theta(3L) = 0$ gives:

$$\begin{aligned} -EI \cdot 0 &= \tfrac{1}{2}F \cdot \langle 2L\rangle^2 - F \cdot \langle L\rangle^2 + \tfrac{1}{2}C_1\,(3L)^2 + 3L \cdot C_2 \\ 0 &= 2FL^2 - FL^2 + \tfrac{9}{2}C_1L^2 + 3C_2L = FL^2 + \tfrac{9}{2}C_1L^2 + 3C_2 \end{aligned}$$

With $w(3L) = 0$:

$$\begin{aligned} EI \cdot 0 &= \tfrac{1}{6}F \cdot \langle 2L\rangle^3 - \tfrac{1}{3}F \cdot \langle L\rangle^3 + \tfrac{1}{6}C_1\,(3L)^3 + \tfrac{1}{2}C_2\,(3L)^2 \\ 0 &= \tfrac{4}{3}FL^3 - \tfrac{1}{3}FL^3 + \tfrac{9}{2}C_1L^3 + \tfrac{9}{2}C_2L^2 = FL^3 + \tfrac{9}{2}C_1L^3 + \tfrac{9}{2}C_2L^2 \end{aligned}$$

where one sees immediately (difference between the two evaluated functions) that $C_2 = 0$, which gives $C_1 = -\frac{2}{9}F$. So the final bend line function is given by:

$$w(x) = \frac{1}{EI}\left[\tfrac{1}{6}F \cdot \langle x - L\rangle^3 - \tfrac{1}{3}F \cdot \langle x - 2L\rangle^3 - \tfrac{1}{27}Fx^3\right]$$

Exercise 3

For the beam with bending stiffness EI in figure 4, calculate the bending line with singularity functions. The distributed force $q_0(x)$ is linearly decreasing from $\frac{2F}{L}$ at point C to point D where it becomes zero.

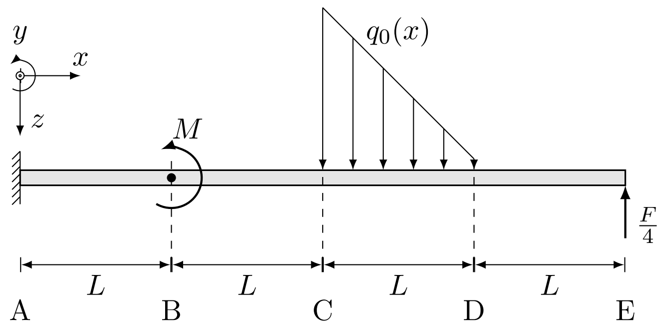

Figure 4: Single clamped beam under complex bending loads.

Exercise solution 3

Given: Geometry, Distributed Force, Moment M, Bending stiffness EI.

Asked: Bending line of the loaded beam.

Relevant relationships: Bending equation

$$EI \cdot w^{(4)}(x) = EI \cdot \frac{\partial^4 w(x)}{\partial x^4} = q(x)$$

with distributed load $q(x)$.

We need to find the distributed load $q(x)$ by summing up all the parts of the distributed loads. The moment in B can be modelled as the first derivative of a Dirac delta (one infinitesimal force acting up, immediately followed by another infinitesimal force acting down). The distributed load is modelled with $\langle x \rangle^1$ functions and a step for the offset at C (see figure 5). The force F at the end does not need to be modelled, it is a boundary condition!

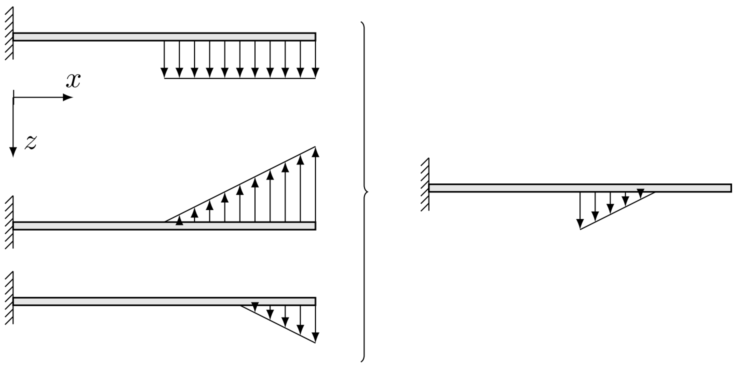

Figure 5: By adding up singularity functions we can model an arbitrary isolated distributed load in the middle of the beam.

All in all we get:

$$q(x) = \underbrace{M \cdot \langle x-L \rangle^{-2}}_{\text{Point Moment}} \underbrace{+\tfrac{2F}{L} \cdot \langle x-2L \rangle^{0} - \tfrac{2F}{L^2} \cdot \langle x-2L \rangle^{1} + \tfrac{2F}{L^2} \cdot \langle x-3L \rangle^{1}}_{\text{Distributed linear force}}$$

For boundary conditions we find that at the end of the beam we have force constraints:

$$V(x=4L) = -\frac{F}{4} \qquad M(x=4L) = 0$$

At the beginning of the beam there are deflection constraints:

$$w(x=0) = 0 \qquad \theta(x=0) = 0$$

So we can start integrating to find the beam deflection equation:

$$\begin{aligned} EI \cdot w^{(4)}(x) &= M \cdot \langle x-L \rangle^{-2} \\ &\quad + \tfrac{2F}{L} \cdot \langle x-2L \rangle^{0} - \tfrac{2F}{L^2} \cdot \langle x-2L \rangle^{1} + \tfrac{2F}{L^2} \cdot \langle x-3L \rangle^{1} \end{aligned} \tag{8}$$

$$\begin{aligned} EI \cdot w^{(3)}(x) &= M \cdot \langle x-L \rangle^{-1} + C_1 + \tfrac{2F}{L} \cdot \langle x-2L \rangle^{1} \\ &\quad - \tfrac{F}{L^2} \cdot \langle x-2L \rangle^{2} + \tfrac{F}{L^2} \cdot \langle x-3L \rangle^{2} = -V(x) \end{aligned} \tag{9}$$

$$\begin{aligned} EI \cdot w^{(2)}(x) &= M \cdot \langle x-L \rangle^{0} + C_1 x + C_2 + \tfrac{F}{L} \cdot \langle x-2L \rangle^{2} \\ &\quad - \tfrac{F}{3L^2} \cdot \langle x-2L \rangle^{3} + \tfrac{F}{3L^2} \cdot \langle x-3L \rangle^{3} = -M(x) \end{aligned} \tag{10}$$

$$\begin{aligned} EI \cdot w^{(1)}(x) &= M \cdot \langle x-L \rangle^{1} + \tfrac{1}{2} C_1 x^2 + C_2 x + C_3 + \tfrac{F}{3L} \cdot \langle x-2L \rangle^{3} \\ &\quad - \tfrac{F}{12L^2} \cdot \langle x-2L \rangle^{4} + \tfrac{F}{12L^2} \cdot \langle x-3L \rangle^{4} = -\theta(x) \end{aligned} \tag{11}$$

$$\begin{aligned} EI \cdot w(x) &= \tfrac{1}{2} M \cdot \langle x-L \rangle^{2} + \tfrac{1}{6} C_1 x^3 + \tfrac{1}{2} C_2 x^2 + C_3 x + C_4 \\ &\quad + \tfrac{F}{12L} \cdot \langle x-2L \rangle^{4} - \tfrac{F}{60L^2} \cdot \langle x-2L \rangle^{5} + \tfrac{F}{60L^2} \cdot \langle x-3L \rangle^{5} \end{aligned} \tag{12}$$

From where we just have to determine the unknown constants C_{1-4}. From $w(x=0)=0$ we directly get $C_4 = 0$ because everything other than C_4 is dependent on x! Analogous we get $C_3 = 0$ for $\theta(x=0)=0$. From $V(x=4L) = -\frac{F}{4}$ in (9) we get:

$$\begin{aligned} \cancel{M \cdot \langle 3L \rangle^{-1}} + C_1 + \tfrac{2F}{L} \cdot \langle 2L \rangle^{1} - \tfrac{F}{L^2} \cdot \langle 2L \rangle^{2} + \tfrac{F}{L^2} \cdot \langle L \rangle^{2} &= +\tfrac{1}{4} F \\ \rightarrow \quad C_1 &= -\tfrac{3}{4} F \end{aligned}$$

The last constant we get from $M(x=4L)=0$, so we use (10) to get:

$$\begin{aligned} 0 &= M \cdot \langle 3L \rangle^{0} - \tfrac{3}{4} F \cdot 4L + C_2 + \tfrac{F}{L} \cdot \langle 2L \rangle^{2} - \tfrac{F}{3L^2} \cdot \langle 2L \rangle^{3} + \tfrac{F}{3L^2} \cdot \langle L \rangle^{3} \\ C_2 &= \tfrac{4}{3} FL - M \end{aligned}$$

So we can combine C_{1-4} into (12) to get the full bending equation:

$$\begin{aligned} w(x) = \frac{1}{EI} \cdot \Big[&\tfrac{1}{2} M \cdot \langle x-L \rangle^{2} - \tfrac{1}{8} F \cdot x^3 + (\tfrac{2}{3} FL - \tfrac{1}{2} M) \cdot x^2 \\ &+ \tfrac{F}{12L} \cdot \langle x-2L \rangle^{4} - \tfrac{F}{60L^2} \cdot \langle x-2L \rangle^{5} + \tfrac{F}{60L^2} \cdot \langle x-3L \rangle^{5} \Big] \end{aligned}$$

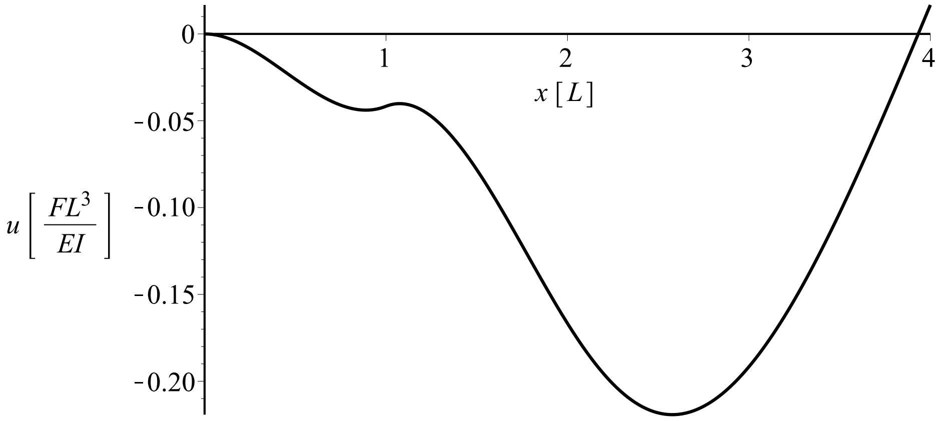

Figure 6: Solved beam line for the given loads if $M = F \cdot L$.

Exercise 4

The supported beam shown in figure 7 is loaded by a uniform load between its two supports and a point force at point C. Using the singularity function method, find the bending moment $M(x)$ and shear force $V(x)$, the bend angle $\theta(x)$ and the deflection $w(x)$ as a function of F.

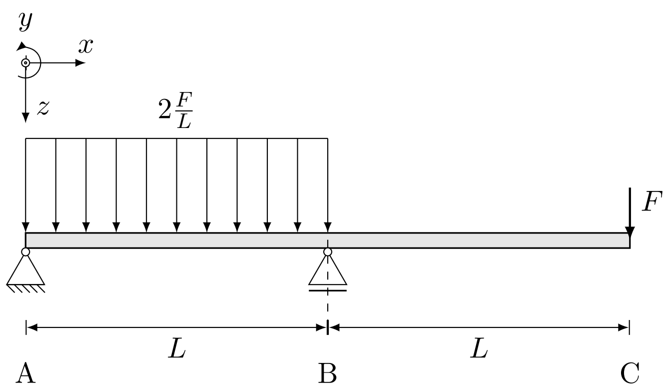

Figure 7: Supported beam with a distributed load and a pointload.

Exercise solution 4

Given: Geometry, loads.

Asked: Shear force and bending moment diagrams.

Relevant relationships:

Singularity function: constant load

$$q(x) = q_0 \langle x - x_0 \rangle^0$$

Singularity function: point load

$$q(x) = F_0 \langle x - x_0 \rangle^{-1}$$

relationship between load q(x), shear force V(x) and bending moment M(x)

$$\frac{dV}{dx} = -q(x), \frac{dM}{dx} = V$$

Integration constants method We show two slightly different ways for solving this problem. The first makes use of integration constants, which are determined in the end, using the boundary conditions. This has the advantage that it also works for over-constrained systems.
The boundary reactions can be neglected, but the center support has to be replaced with an (unknown) force R_B. The load function becomes:

$$q(x) = \frac{2F}{L}\left(1 - \langle x - L \rangle^0\right) - R_B \langle x - L \rangle^{-1}$$

Going through the chain of integrations gives:

$$\begin{aligned}
-V = EIw^{(3)} &= \frac{2F}{L}\left(x - \langle x - L \rangle^1\right) - R_B \langle x - L \rangle^0 + C_1 \\
-M = EIw^{(2)} &= \frac{F}{L}\left(x^2 - \langle x - L \rangle^2\right) - R_B \langle x - L \rangle^1 + C_1 x + C_2 \\
-EI\theta = EIw' &= \frac{F}{3L}\left(x^3 - \langle x - L \rangle^3\right) - \frac{R_B}{2} \langle x - L \rangle^2 + \frac{C_1}{2}x^2 + C_2 x + C_3 \\
EIw &= \frac{F}{12L}\left(x^4 - \langle x - L \rangle^4\right) - \frac{R_B}{6} \langle x - L \rangle^3 + \frac{C_1}{6}x^3 + \frac{C_2}{2}x^2 \\
&\quad + C_3 x + C_4
\end{aligned}$$

where we immediately get $C_2 = 0$ because $M(0) = 0$ and $C_4 = 0$ due to $w(0) = 0$. Evaluating the free end with force gives the boundaries $V(2L) = F$ and $M(2L) = 0$:

$$\begin{aligned}
F &= -2F + R_B - C_1 \\
0 &= 3FL - R_B L + 2C_1 L
\end{aligned}$$

which solves to $C_1 = 0$ and $R_B = 3F$. Finally we use $w(L) = 0$ (pin support in the middle) to get:

$$0 = \frac{F}{12}L^3 + C_3 L \quad \rightarrow \quad C_3 = -\frac{FL^2}{12}$$

So finally:

$$
\begin{aligned}
V &= -\frac{2F}{L}\left(x - \langle x-L\rangle^1\right) + 3F\langle x-L\rangle^0 \\
M &= -\frac{F}{L}\left(x^2 - \langle x-L\rangle^2\right) + 3F\langle x-L\rangle^1 \\
\theta &= \frac{1}{EI}\left[\frac{F}{3L}\left(x^3 - \langle x-L\rangle^3\right) - \frac{3F}{2}\langle x-L\rangle^2 + \frac{FL}{12}\right] \\
w &= \frac{1}{EI}\left[\frac{F}{12L}\left(x^4 - \langle x-L\rangle^4\right) - \frac{F}{2}\langle x-L\rangle^3 - \frac{FL^2}{12}x\right]
\end{aligned}
$$

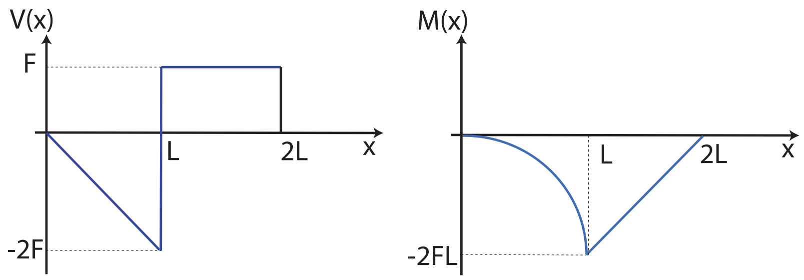

Figure 8: Shear force and bending moment profiles.

Predetermined reactions method Alternatively, we can first calculate all reaction forces/moments and add them as point loads/moments in the load equation.

First we need to find all forces including reaction forces at points A and B. In order to find R_A and R_B, we write the force equilibrium in the bar and also moment equilibrium at point A. We substitute the uniform load profile from 0 to L with its equivalent point fore $2F$ at $\frac{L}{2}$.

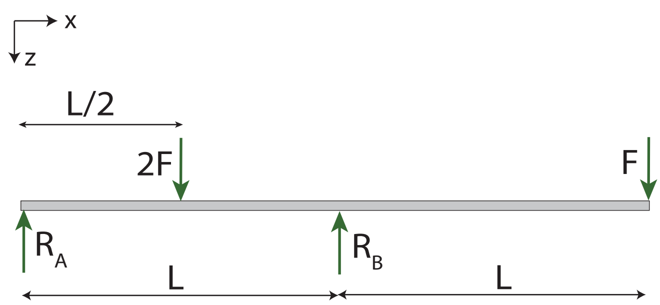

Figure 9: Equivalent forces acting on the beam.

Force equilibrum

$$\Sigma F_z = 0$$
$$\frac{2F}{L} \cdot L + F - R_A - R_B = 0 \quad \rightarrow \quad R_A + R_B = 3F$$

Moment equilibrium at point A

$$\Sigma M_A = 0$$
$$2F \times \frac{L}{2} - R_B \cdot L + F \times 2L = 0 \quad \rightarrow \quad R_B = 3F$$
$$R_A + R_B = 3F \quad \rightarrow \quad R_A = 0$$

Now we can write the load $q(x)$ along the bar using singularity functions:

$$q(x) = \frac{2F}{L}\langle x\rangle^0 - \frac{2F}{L}\langle x-L\rangle^0 - R_A\langle x\rangle^{-1} - R_B\langle x-L\rangle^{-1}$$
$$\rightarrow \quad q(x) = \frac{2F}{L}\langle x\rangle^0 - \frac{2F}{L}\langle x-L\rangle^0 - 3F\langle x-L\rangle^{-1}$$

We don't need any integration constants for the shear force and moment, since all the reactions are already modeled in the distributed load. By integrating $q(x)$ we find $V(x)$:

$$V(x) = -\frac{2F}{L}\langle x\rangle^1 + \frac{2F}{L}\langle x-L\rangle^1 + 3F\langle x-L\rangle^0$$

Now we find the bending moment $M(x)$ by taking the integration from shear force $V(x)$:

$$M(x) = -\frac{2F}{L}\cdot\frac{1}{2}\langle x\rangle^2 + \frac{2F}{L}\cdot\frac{1}{2}\langle x-L\rangle^2 + 3F\langle x-L\rangle^1$$
$$= -\frac{F}{L}\langle x\rangle^2 + \frac{F}{L}\langle x-L\rangle^2 + 3F\langle x-L\rangle^1$$

Integrating this twice more gives the bending angle and the bend line (deflection) respectively. Note that here integration constants are required!

$$EI\theta(x) = -\frac{F}{3L}\langle x\rangle^3 + \frac{F}{3L}\langle x-L\rangle^3 + \frac{3F}{2}\langle x-L\rangle^2 + C_1$$
$$EIw(x) = \frac{F}{12L}\langle x\rangle^4 - \frac{F}{12L}\langle x-L\rangle^4 - \frac{F}{2}\langle x-L\rangle^3 - C_1x + C_2$$

Where $C_2 = 0$ because $w(0) = 0$. The last constant we get with $w(L) = 0$ (no deflection at the central support):

$$0 = \frac{FL^3}{12} - C_1L \quad \rightarrow \quad C_1 = \frac{FL^2}{12}$$

And we finally get

$$\theta(x) = \frac{1}{EI}\left[-\frac{F}{3L}\langle x\rangle^3 + \frac{F}{3L}\langle x-L\rangle^3 + \frac{3F}{2}\langle x-L\rangle^2 + \frac{FL^2}{12}\right]$$
$$w(x) = \frac{1}{EI}\left[\frac{F}{12L}\langle x\rangle^4 - \frac{F}{12L}\langle x-L\rangle^4 - \frac{F}{2}\langle x-L\rangle^3 - \frac{FL^2}{12}\cdot x\right]$$