# Exercise 1

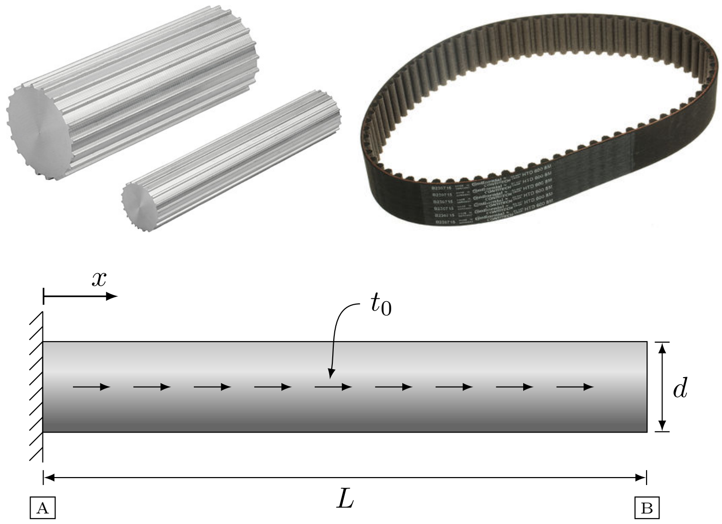


Figure 1: Bar loaded with a distributed torque. Such a load can arise for example from a broad belt driving a shaft.

Transmission belts are a way to transmit power from one shaft to another. The torque in that case is no longer a point torque applied to a bar, but is distributed across the length of the bar. A round shaft (see figure 1) with diameter $d$ and length $L$ is fixed on one side. A distributed torque $t_0 = T_0/L$ is applied on the whole length of the shaft. The shaft is made of a material with known shear modulus $G$.
Calculate and sketch the internal moment $T(x)$ and the angle of twist $\varphi(x)$ over the length of the shaft.

# Exercise solution 1

**Given:**

- Diameter of the bar: $d$
- Bar length: $L$
- Shear modulus of bar: $G$
- Distributed load $t_0 = T_0/L$

**Asked:**

- Internal moment $T(x)$ along the bar.
- Twist $\varphi(x)$ along the bar.

**Relevant relationships:**

- Second moment of inertia:

$$J = \int r^2 \,\mathrm{d}A = \frac{\pi r^4}{2}$$

- Total moment from distributed moment between $a$ and $b$:

$$T_{ab} = \int_a^b t(x)\,\mathrm{d}x$$

- Angle of twist:

$$\varphi = \int_0^L \frac{T(x)}{G(x)J(x)}$$

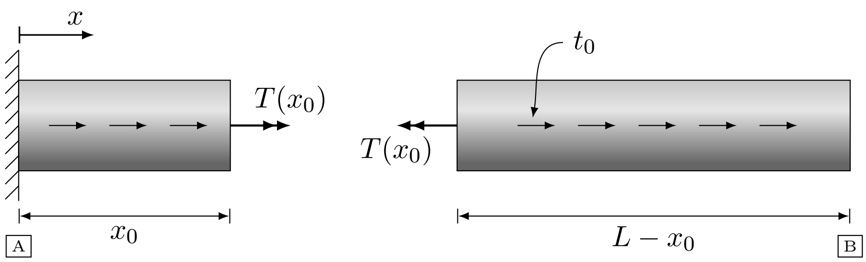


Figure 2: Section through bar for internal moment.

We get the internal torque with the moment of sections (see figure 2) by equilibrium of moments on the free part as:

$$-T(x_0) + \int_{x_0}^{L} t(x)\,\mathrm{d}x = -T(x_0) + \int_{x_0}^{L} \frac{T_0}{L}\,\mathrm{d}x = -T(x_0) + \frac{T_0}{L}(L - x_0) = 0$$

and so

$$T(x_0) = \frac{T_0}{L}(L - x_0)$$

and using the formula for the twist as function of the $x_0$ position

$$\varphi(x_0) = \int_0^{x_0} \frac{T(x)}{GJ}\,\mathrm{d}x = \frac{1}{GJ}\int_0^{x_0} \frac{T_0}{L}(L - x)\,\mathrm{d}x = \frac{T_0}{GJ} \cdot \left(x_0 - \frac{x_0^2}{2L}\right)$$

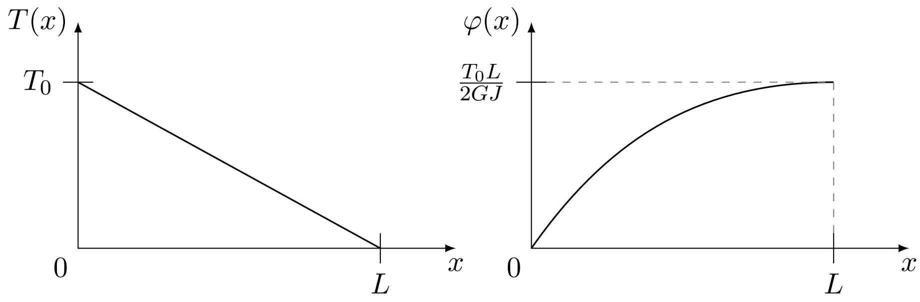


Figure 3: Plots of the torque and the angle of twist along the beam.

## Exercise 2

You have a football made of cowhide (Young's Modulus: $E = 7000PSI$). Its diameter is $d = 8.6inches$ and the specification reads that it should be inflated to $p_{norm} = 0.9Bar$.

- Calculate the hoop stress in the leather if its thickness is $t = 2mm$.
- The yield strength of cowhide is $\sigma_{yield} = 20N/mm^2$. What is the maximum allowable pressure inside the ball? What is the safety factor when the ball is inflated to the norm? Assume that the ball's diameter and the leather thickness remain constant.

## Exercise solution 2

First thing to do: change all units to something understandable!

$$E = 7000PSI = 48.3MPa$$

$$d = 8.6inches = 21.84cm$$

$$p_{norm} = 0.9Bar = 90kPa$$

$$\sigma_{yield} = 20N/mm^2 = 20MPa$$

### a)

$$\sigma_{hoop} = \frac{p \cdot r_i}{2t} = 2.46MPa$$

### b)

$$p_{max} = \frac{2 \cdot t \cdot \sigma_{yield}}{r_i} = 732.6kPa = 7.3Bar$$

$$SF = \frac{p_{max}}{p_{norm}} = \frac{\sigma_{max}}{\sigma_{hoop}} \approx 8$$

## Exercise 3

A thin-walled cylindrical pressure vessel with a circular cross section is subjected to internal gas pressure $p$ and simultaneously elongated by an axial load $F = 1.5\,\mathrm{N}$. The cylinder has inner radius $r_1 = 2.1\,\mathrm{mm}$ and outer radius of $r_2 = 2.25\,\mathrm{mm}$. Determine the maximum allowable internal pressure if the ultimate stress of the material is $1\,\mathrm{MPa}$.

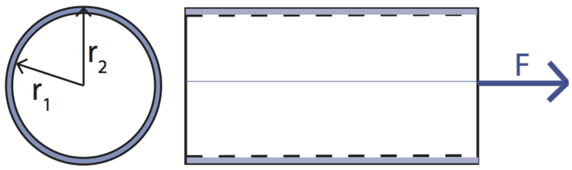


Figure 4: Pressure vessel.

## Exercise solution 3

**Given:** inner and outer radii, axial load $F = 1.5\,\mathrm{N}$, ultimate stress=$1\,\mathrm{MPa}$.

**Asked:** maximum allowable internal pressure p

**Relevant relationships:**

*Longitudal and Hoop stress*

$$\sigma_L = \frac{pr}{2t}$$

$$\sigma_H = \frac{pr}{t}$$

There are two forces acting along the pressure vessel: one comes from the internal pressure $p$ and the other is due applied axial load $F$. Considering $t = r_2 - r_1$:

$$\sigma_L = \frac{pr_1}{2t}$$

$$\sigma_P = -\frac{F}{2\pi r_1 t}$$

$$\sigma_x = \sigma_L + \sigma_P = \frac{pr_1}{2t} + \frac{F}{2\pi r_1 t}$$

and Hoop stress acts radially:

$$\sigma_H = \sigma_y = \frac{pr_1}{t}$$

The ultimate stress is 1 MPa, so normal stresses $\sigma_x$ and $\sigma_y$ should be less than 1 MPa:

$$\sigma_x = \sigma_L + \sigma_P = \frac{pr_1}{2t} + \frac{F}{2\pi r_1 t} = \frac{p \cdot 2.1 \times 10^{-3}}{2 \cdot 0.15 \times 10^{-3}} + \frac{1.5}{2\pi \cdot 2.1 \times 10^{-3} \cdot 0.15 \times 10^{-3}}$$

$$= 7p + 0.758 \times 10^6 < 1\,\text{MPa} \quad \rightarrow \quad p < 34.6\,\text{kPa}$$

Hoop stress:

$$\sigma_y = \frac{pr_1}{t} = 14p < 1\,\text{MPa} \quad \rightarrow \quad p < 71.4\,\text{kPa}$$

so the internal pressure should not exceed 34.6 kPa.