

Exercise 1

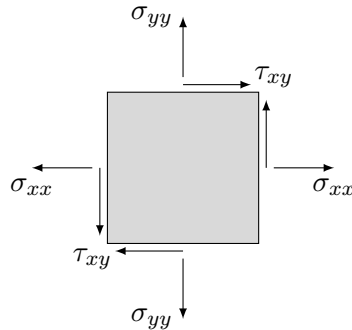


Figure 1: The state of stress on an element of a structure

A 2D stress element is subjected to the normal stresses $\sigma_{xx} = 50$ MPa, $\sigma_{yy} = 10$ MPa and shear stress $\tau_{xy} = -20$ MPa shown in the drawing in figure 1. We know that the material of the element has a weak axis rotated 30° counter clockwise.

- a) What are the normal and shear stresses along that axis? Calculate once with the formulas we derived in class and once with matrix rotation.

As discussed in class, there exists for every stress state a set of directions in which the normal stresses are maximum and minimum, and the shear stresses are equal to zero. These axes are called the principal axes, and the corresponding stresses are called the principal stresses.

- b) For the stress state above, calculate the principal stresses and the principal axes using the formulas from the formula sheet.
- c) Calculate the principal stresses and the principal axes of the element using the stress tensor. *Hint: the principal stresses are the eigenvalues of the stress tensor, while the principal axes are given by the eigenvectors.*

Exercise 2

In a gram-negative bacterium, the peptidoglycan layer is situated between the inner and outer lipid bi-layer. This peptidoglycan layer is believed to be responsible for the mechanical stability and robustness of the bacterial wall. Figure 2a depicts the architecture of the peptidoglycan, which a recent study has revealed¹.

We simplify the shown peptidoglycan architecture and consider the model as shown in figure 2b. The bacterium has an internal pressure p of 150 kPa. The

¹Hayhurst, Emma J., et al. "Cell wall peptidoglycan architecture in *Bacillus subtilis*." Proceedings of the National Academy of Sciences 105.38 (2008): 14603-14608.

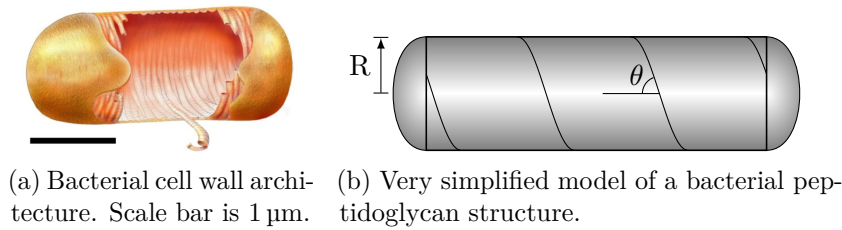


Figure 2: Gram-negative bacterium.

peptidoglycan cables form an angle $\theta = 55^\circ$ with the longitudinal axis of the cylinder. Consider only the peptidoglycan layer as the bacterial wall with a thickness of $t = 50 \text{ nm}$, a Young's modulus $E = 10 \text{ MPa}$ and a Poisson ratio of $\nu = 0.3$. The rod shaped bacterium has a radius of $R = 0.5 \mu\text{m}$. Calculate:

- The principal stresses in an element in the cylindrical part of the bacterium. To do so, consider a 2D element of the bacterial wall, assuming plane stress.
- The tensile and shear stresses in an element aligned to the peptidoglycan cables.
- The longitudinal and hoop strains in the bacterium.

Hint: Use the approximations for stress in thin walled structures.

Exercise 3

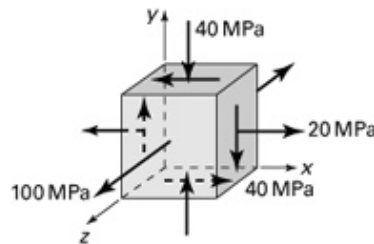


Figure 3: The state of stress on an element of a structure

The state of stress on an element of a structure is illustrated in Figure 4.

- Determine the principal stresses σ_1 , σ_2 and σ_3 .
- Deduce from it the maximum shear stress $\tau_{\max}$, knowing that

$$\tau_{\max} = \frac{\max(\sigma_1, \sigma_2, \sigma_3) - \min(\sigma_1, \sigma_2, \sigma_3)}{2}$$

Exercise 4

A wire strain gauge can effectively measure strain in only one direction. To determine the three independent components of plane strain, three linearly independent strain measures are needed, i.e., three strain gauges positioned in a rosette-like layout.

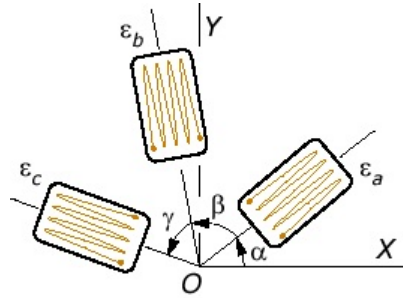


Figure 4: Rosette-like strain layout

- Determine the value of ϵ_a , ϵ_b and ϵ_c as a function of ϵ_x , ϵ_y , γ_{xy} , α , β and γ . You can derive the transformation formulas for strain in analogy to the stress transformations.
- For $\alpha = 30^\circ$ and $\beta = \gamma = 60^\circ$ (60° strain rosette configuration), you measure the following strains : $\epsilon_a = 300\mu m/m$, $\epsilon_b = 100\mu m/m$ and $\epsilon_c = 10\mu m/m$. Determine the values of ϵ_x , ϵ_y and γ_{xy} .