Exercise 1

Many engineering materials at the macro-scale can be considered linear-elastic, but this kind of approximation often breaks down when we look at objects at very small length scales. For example, a molecule of DNA in solution will naturally collapse into a random coil, and require a force to stretch it out. One model for the restoring force F of a DNA molecule whose endpoints are stretched apart a distance x is given by:

$$F = \left(\frac{k_B T}{L_p}\right)\left[\frac{1}{4\left(1-\frac{x}{L_0}\right)^2} - \frac{1}{4} + \frac{x}{L_0}\right] \tag{1}$$

where k_B is the boltzman constant, T is the absolute temperature, L_p is the persistence length of the DNA (a measure of the resistance of the DNA molecule to bending), and L_0 is the contour length (the length along the path of the DNA). The manipulation of DNA is very important in biology. It must be packaged into a compact structure during cell division, and for a gene to be read, the section of interest must be unpackaged into a more stretched-out configuration.

a) Calculate the stored energy of a DNA molecule stretched from 0 to a distance L which is less than L_0.

b) What amount of energy is needed to stretch a DNA molecule with $L_0 = 2\,\mu\text{m}$ from 0 to a length of $1.9\,\mu\text{m}$?

   Use a persistence length of $50\,\text{nm}$, a temperature of $37\,°\text{C}$, and $k_B = 1.38 \cdot 10^{-23}\text{J}\,\text{K}^{-1}$.

Exercise 2

Normal and shear strains are described by the following displacement field:

$$u\left(x,y,z\right) = -z\frac{\partial f(x)}{\partial x}, \qquad v\left(x,y,z\right) = 0, \qquad w\left(x,y,z\right) = f(x) \tag{2}$$

where

$$f(x) = \frac{L^3}{6}\left(3\left(\frac{x}{L}\right)^2 - \left(\frac{x}{L}\right)^3\right) \tag{3}$$

and L is a constant. Calculate all of the normal and shear strains for the defined displacement field and write the 3D strain tensor.

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Note Function $f(x)$ multiplied by some factor actually presents the deflection of the cantilever beam induced by the point load. Derived strain ε_x is the induced strain in the beam along the cantilever length. But all of this you will learn soon... :)

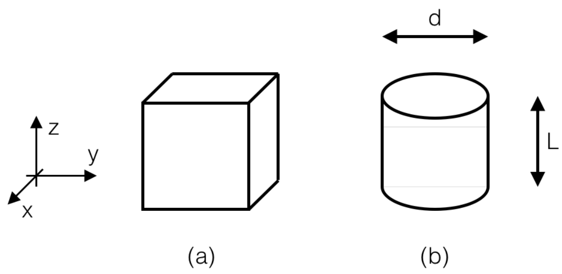

Figure 1: Definition of the geometrical entities.

Exercise 3

a) The normalized volume change of a parallelepiped as a function of the strain in all three directions:

$$\frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Prove this expression by considering a parallelepiped with sides x, y and z (fig. 1, a). Hint: you will have to cross-out negligible terms.

b) Similarly to part (a), show that a cylinder (fig. 1, b) that is strained in both the radial and the axial direction will experience a volume change described by:

$$\frac{\Delta V}{V_0} = \varepsilon_L + 2\varepsilon_r$$

c) When a woman is standing on her two feet, she puts a weight of about w = 200N on each of her femurs. By approximating the bone as a cylinder of diameter d = 2cm and length L = 40cm, find its change in diameter when the woman goes from a sitting to a standing position. Assume that the volume of the bone remains constant.

Young's modulus for the bone: E = 16GPa

Exercise 4

A block of rubber (R) is confined in a slot inside a steel block (S). A uniform pressure p_0 to the top of the rubber block induces a deformation.

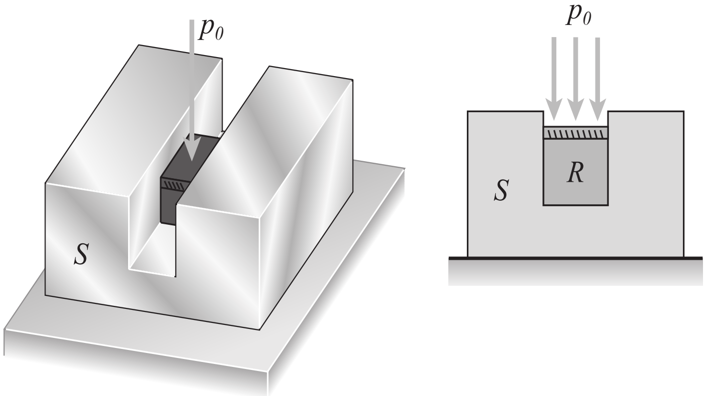

Figure 2: Rubber block in a slotted steel piece.

a) Find a formula for the lateral pressure on the block, induced by the downward pressure p_0, ignoring any friction effects.

b) Derive a formula for the dilation e of the rubber (the volume change ratio $\Delta V/V$).

Exercise 5

A rubber cube with side length $a = 1cm$ is placed on a flat surface (see figure 3). A weight of mass $m = 20g$ is added on top. The cube has a known Young's modulus $E = 0.1GPa$, poisson ratio $\nu = 0.45$.
You will neglect the variation of area in the calculation of the stress : $\sigma \approx \frac{F}{A_{\text{no stress}}}$ (as you did up to now in this course).

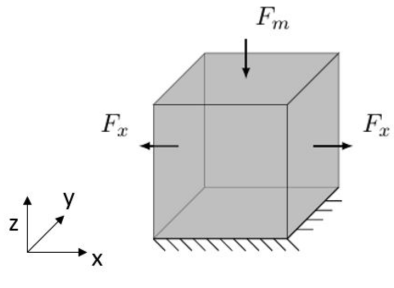

Figure 3: Rubber block on surface.

a) What is the change in height of the cube Δz? What is the change of volume ΔV?

b) You now want to bring back the cube to its original height. Which force F_x do you need to apply? What is the change of volume ΔV?

c) Instead, you want to bring the cube back to its original volume of a^3. Which force F_x do you need to apply ?

d) The force F_x is no longer applied, only the force F_m due to the weight remains. Use calculation at the first order (small deformations) to determine the deformation ε_z and the stress σ_z without making the approximation that $\sigma \approx \frac{F}{A_{\text{no stress}}}$. Instead, use: $\sigma = \frac{F}{A_{\text{with stress}}}$. Compare to your results with the approximation (first question). Conclude : is this approximation reasonable?