Exercise 1

The mechanical properties of a healed femur bone are being investigated. In the place where the break healed, a thickening of the bone has occurred. A simplified sketch of the structure can be seen in figure 1. The bone has a uniform Young's modulus of $E_1 = 60\,\text{GPa}$ in the unbroken part and $E_2 = 40\,\text{GPa} = \frac{2}{3}E_1$ in the healed part, a segment length of $L = 20\,\text{cm}$ and a cross sectional area of $A = 5\,\text{cm}^2$.

If the bone is clamped at both ends and stressed by a force $F = 11\,\text{kN}$ at the thickened part, calculate the external reaction forces and displacements. Use the displacement stiffness method.

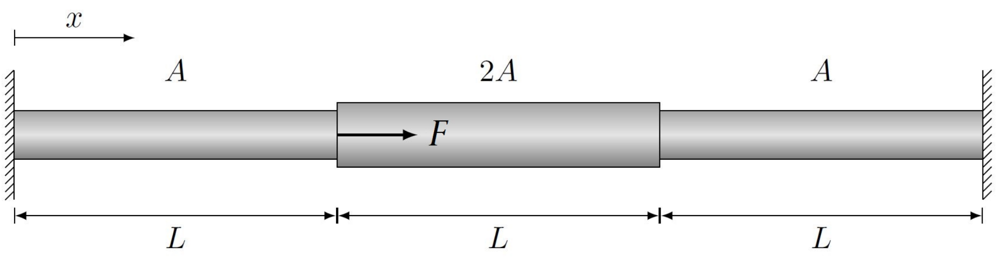


Figure 1: Simplified structure of a bone with a fortification in the middle that is being loaded.

Exercise 2

A force F acts at point B on a structure (see Fig. 2). The structure is clamped on both sides and the maximum diameter of the structure is d_A while the small diameter is d_B. The structure has a Young's modulus E and a length 2L.

a) Define the nodes in the system. Deduce the displacement-stiffness equation for the system. You will use k_1 as the stiffness of the left part of the system (between points A and B) and k_2 as the stiffness of the right part of the system (between points B and C), without trying to find their value yet.

b) For this question and the next one, we consider that $d_A = 2d$ and $d_B = d$. Determine k_1 and k_2 as a function of E, L and d.

c) Deduce the reaction forces R_A and R_C, as well as the displacement u_B, as a function of E, L, d and F. Draw the reaction forces on the schematics and respect the sign convention.

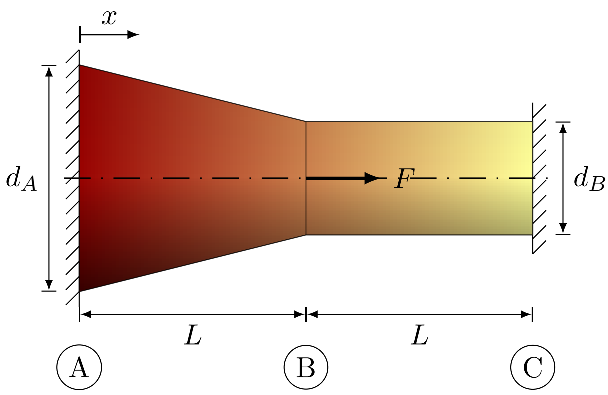


Figure 2: Force F acting on the structure.

Exercise 3

A prosthesis part made out of an aluminum alloy has the geometry as depicted in figure 3. The widths are $b = 60$ mm, $c = 30$ mm, $d = 5$ mm and the thickness is $t = 5$ mm. The fillets have a radius of $R = 8$ mm. The ultimate strength of the aluminum alloy is $\sigma_U = 410\,\text{MPa}$. The fillets and the hole are spaced by $80mm$.
For both questions, you can use figures 4 and 5 to determine the stress-concentration factor K.

a) If a factor of safety of SF=2.5 is required, what is the maximum allowable tensile load P_a?

b) *(difficult question, won't be asked at the exam)* Find the hole diameter d_max at which the two segments of the bar have the same tensile load carrying capacity.

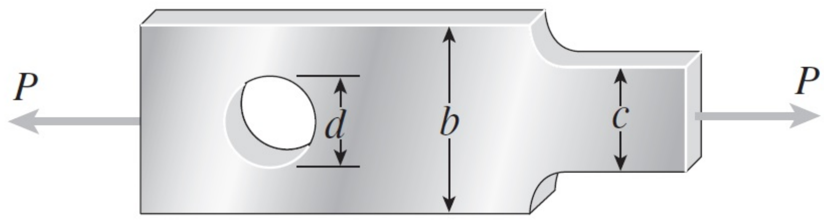


Figure 3: A prosthesis part with a hole.

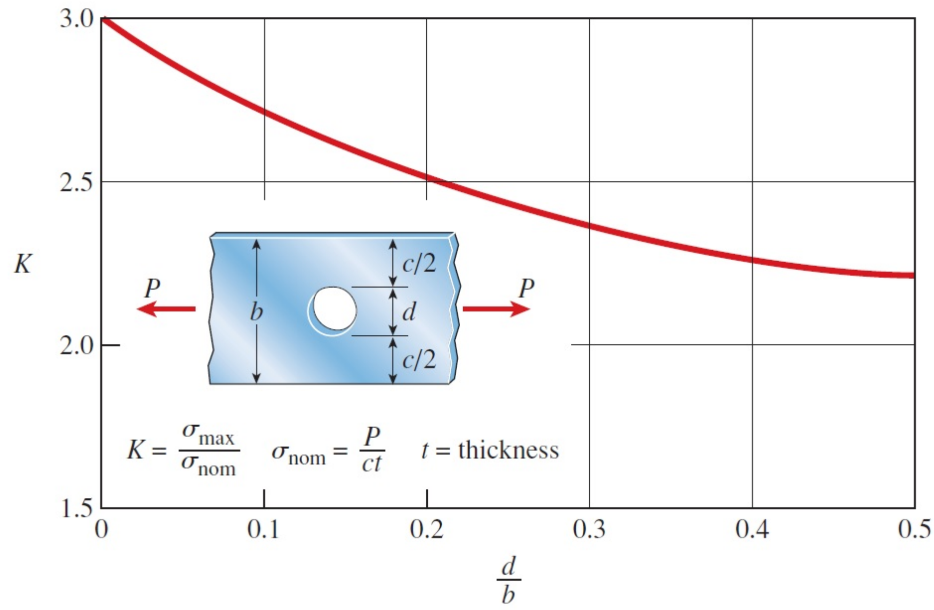


Figure 4: Stress-concentration factor K for flat bars with a circular hole.

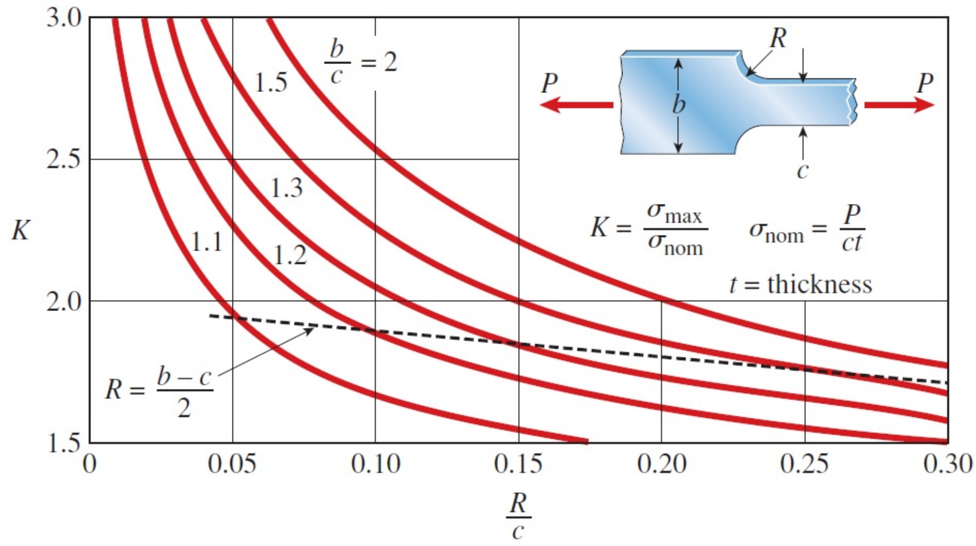
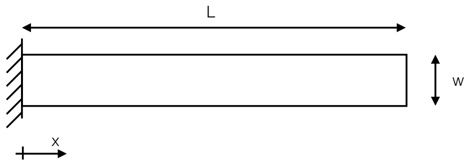


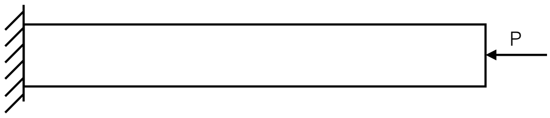
Figure 5: Stress-concentration factor K for flat bars with shoulder fillets.

Exercise 4

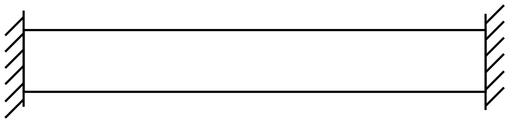
We consider heated bars in different configurations.



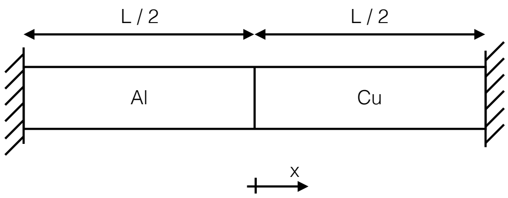
(a) Dimensions of the aluminum bar.



(b) A compressive force P is applied at the open end.



(c) The bar is clamped on both sides.



(d) Dimensions of the Aluminum-Copper bar.

Figure 6: Schematics of the bars.

a) A simple aluminum bar (Young's Modulus $E_{Al} = 69\text{GPa}$, linear temperature expansion coefficient $\alpha_{Al} = 24{\cdot}10^{-6}\text{K}^{-1}$) with a square-cross section is clamped at one end (fig. 6a). At room temperature $T_{room} = 22°\text{C}$, its length is $L = 30\text{cm}$ and its width $w = 5\text{cm}$.

   Calculate the elongation of the bar if the air conditioning fails and the temperature increases to $T_{summer} = 37°\text{C}$.

b) The bar is back at room temperature $T_{room} = 22°\text{C}$. A force P is applied at the open end (fig. 6b).

   Give an expression for the total shortening of the bar as a function of P.

c) The bar is now clamped on both sides.

   Calculate the stress in the bar and the reaction forces at both ends when the temperature increases to $T_{summer} = 37°\text{C}$.

d) The aluminum bar is replaced by a bi-material model. While the external dimensions are the same, it consists of two half-bars of length $L/2$, one of aluminum and the other of copper ($E_{Cu} = 117\text{GPa}$, $\alpha_{Cu} = 16{\cdot}10^{-6}\text{K}^{-1}$).

Find the displacement of the aluminum-copper intersection as well as the stress in the bar when the ambiant temperature increases by $\Delta T = 15°\text{C}$.