Exercise 1

A bar of unstretched length l_0 with uniform mass density ρ is hung from a rigid ceiling. The bar has a cross-sectional area $A(y)$ which varies along the length of the bar and has a value A_0 at the bottom (see figure 1).

a) What is the equation that describes $A(y)$ if the bar is to have uniform stress σ in the horizontal plane along its length?

b) What is the total elongation of the bar in this case?

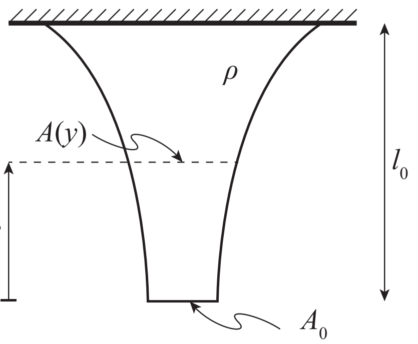

Figure 1: Structure with nonuniform cross–section supported on ceiling.

Exercise 2

You want to calculate the deformation of a Y shaped trabecula section in the trabecular bone of a vertebra as shown in figure 2. To simplify the calculation, we model the Y-shaped trabecula as shown in figure 3. Assume that the horizontal beam is both infinitely thin and stiff and that it does not bend.

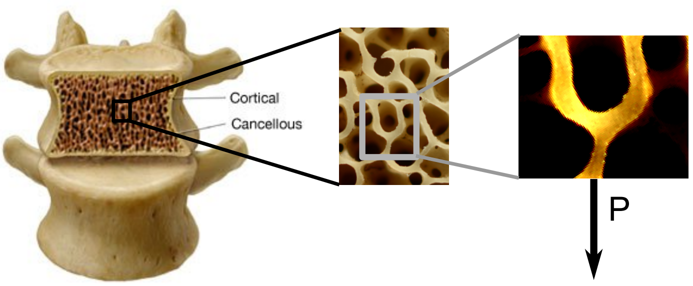

Figure 2: Schematics of a trabecular bone.

1. A force of $F1 = 0.5\,\mathrm{N}$ is applied to the trabecular bone substructure. Calculate the total elongation of the substructure, given the lengths $L_1 = 1.5\,\mathrm{mm}$, $L_2 = 0.8\,\mathrm{mm}$ and the diameter $d = 200\,\mu\mathrm{m}$. Young's modulus of the trabecular bone can be assumed to be $E = 22\,\mathrm{GPa}$.

2. There is now an extra force $F2 = 0.2\,\mathrm{N}$ applied to the bone as shown in figure 3. State the superposition principle and calculate the total elongation.

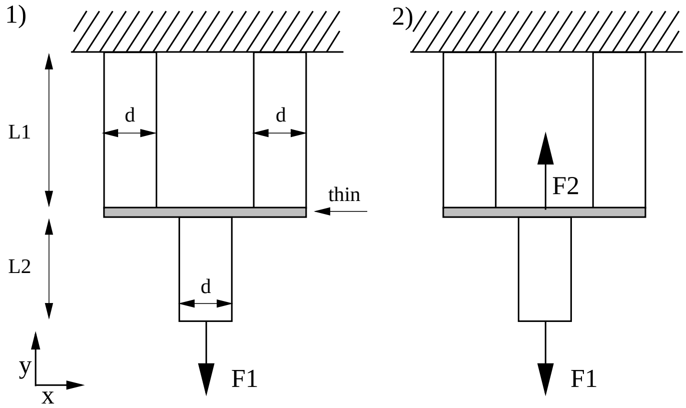

Figure 3: Simplification of the bone for calculations.

Exercise 3

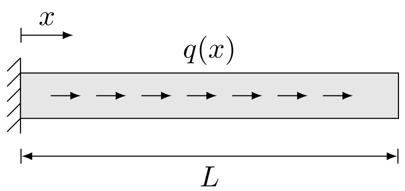

Figure 4: Beam with a distributed load.

The bar in figure 4 is loaded with a force that is distributed over the length of the beam. The load is described as

$$q(x) = q_0 \cdot \frac{x}{L} + q_1$$

and we want to calculate the internal forces $N(x)$ and the displacement field $u(x)$ along the beam.

1. Find the differential equation that describes the displacement field of the beam $u(x)$ as a function of E, A and $q(x)$, by considering the three

essential equations of structural mechanics:
- constitutive equation : $E = \frac{\sigma}{\varepsilon}$
- kinematic equation : $\varepsilon(x) = \dfrac{\partial u(x)}{\partial x}$
- equilibrium equation : $\dfrac{\partial N(x)}{\partial x} + \sum_i q_i(x) + B_x A(x) = 0$

2. Find the boundary conditions for the bar and deduce boundary conditions for u (or its derivatives).

3. Solve the equations for $u(x)$. Deduce the expression of the internal force in the beam $N(x)$.