Exercise 1

Goals of the exercise: (1) understand the difference between macroscopic and microscopic strain, (2) understand qualitatively the relationship between the local deformation $u(x)$ and the local strain $\varepsilon(x)$.

a) We consider the bar in figure 1 before and after applying a force in the longitudinal axis. Use a ruler to measure the distances on the figure and answer the following questions.

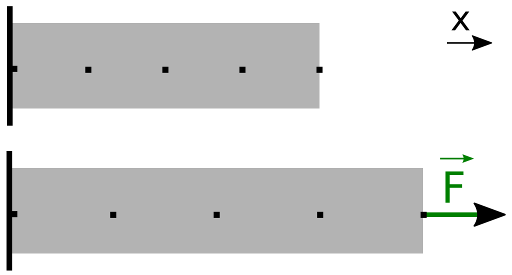

Figure 1: Longitudinal deformation of a rectangular bar

1. What is the value of the macroscopic strain ?
2. Represent on the figure the vector $\vec{u}$ for the third drawn point. What is the displacement $u(x)$ of each of the drawn points ? Plot $u(x)$ on a graph (you can do a linear interpolation between the points).
3. Determine the value of the strain in each individual block (between two points) using the definition of strain ($\varepsilon = \frac{\Delta L}{L}$) and plot it. Show that you get the same answer as with the formula $\varepsilon(x) = \frac{du}{dx}(x)$ demonstrated during the class.
4. What is the relationship between the macroscopic strain and the microscopic strain ?

b) Same questions for the object in figure 2. Conclusion : what are the links and differences between ε, u, the macroscopic strain and the object's total elongation?

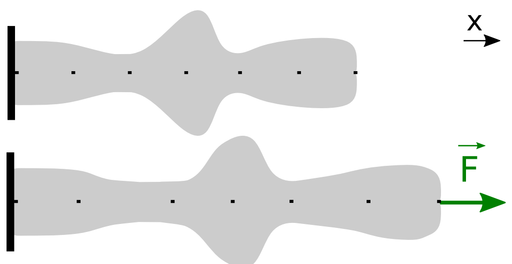

Figure 2: Longitudinal deformation of a free form object

Exercise 2

The crane structure shown in figure 3 is built with individual bars on rotating hinges (white dots). Vertical and horizontal bars are of length L, diagonal ones of length $\sqrt{2}L$. The crane is supported in point A with a fixed hinge (supports forces in x and in y direction) and in point B with a sliding hinge (supports forces in y direction only). The force $\vec{F}_C = (1, -3) \cdot F_c$ is acting on the point C. We will neglect the weight of the crane.

a) Cut the system free (replace the hinges in A and B with replacement forces).

b) Calculate the reaction forces in A and B as function of F_C.

c) Calculate the internal reaction forces in the beams 2–4, 12 and 13.

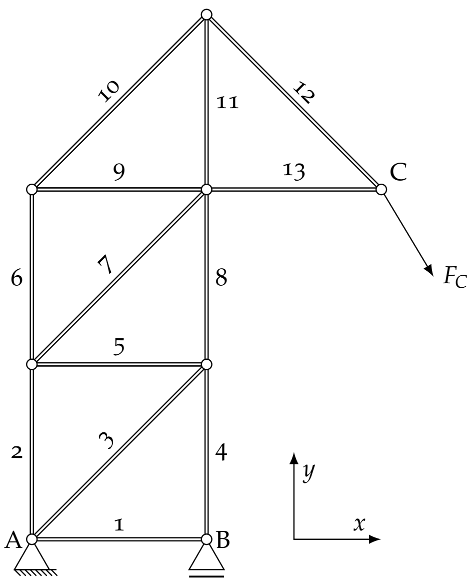

Figure 3: Crane structure with individual bars on rotating hinges .

Exercise 3

A force $P = 1\,\mathrm{kN}$ is applied on a human femur bone (see figure 4(a)). The bone is modeled as a hollow tube with circular cross section and a constant wall thickness of $0.5\,\mathrm{cm}$. The shape of the shaft of the bone is approximated by the quadratic function

$$y = \frac{x^2}{100\,\mathrm{cm}} + 2\,\mathrm{cm}$$

where the origin of x is in the middle of the bone (see figure 4(b)).
For the simplified model, find:

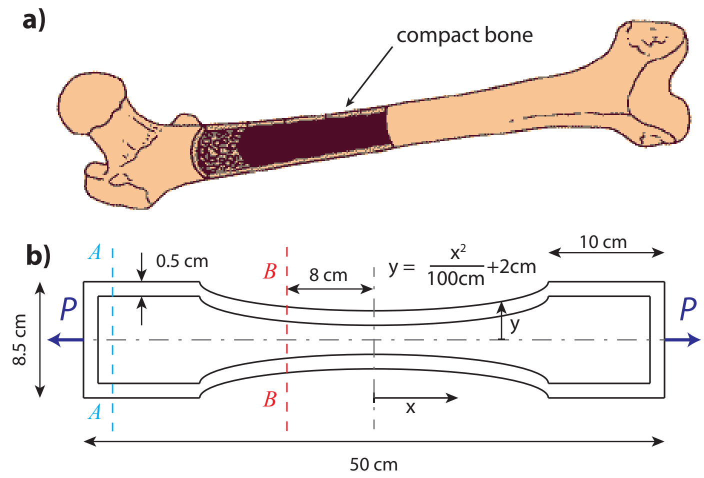

Figure 4: Illustration of human femur bone. a) Sketch of real bone, showing the compact bone wall. b) Sketch of the simplified model of the bone.

a) The stress in the cross–section A.

b) The stress in the cross–section B.

c) Where is the highest stress in the bone and what is its value ? If the load is increased, at which position will the bone break ?

Exercise 4

We write the year 1723 and a young soldier who lost his right leg in battle a few months ago is now sitting in front of you. You are the physician who is charged with designing a wooden leg in order to help him walk around again. Or hobble around, that is. As you don't have much experience with prosthetics, you will just have to try what you think is best.

a) Your first design is to attach a simple wooden stick to his leg (see figure 5). He weighs $m_2 = 80\,\text{kg}$ but used to weigh $m_1 = 88\,\text{kg}$ before his injury. He is still $s = 1.75\,\text{m}$ tall. The diameter of the stick is $d = 2\,\text{cm}$. Calculate the stress σ_z in the contact area between the leg and the stick.

b) A few hours later, your patient comes back because he feels pain in the area where the prosthesis is attached to his knee. You realize then that the skin can only bear a compressive stress of $\sigma_{skin} = 100\,\text{kPa}$ without pain. What is the diameter of the minimum contact area between the leg and the wooden prosthesis that you should use?

c) The wood you chose is of superior quality and can withstand a maximum compressive stress of $\sigma_{wood} = 1.5\,\text{MPa}$. Knowing this, draw a wooden leg that will be both comfortable and light-weight.

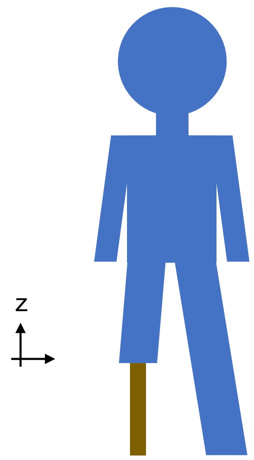

Figure 5: Illustration of the unfortunate soldier with his wooden leg.

Exercise 5

A force P is acting at the free end of a micro bar and we are measuring the resulting elongation of the bar caused by this force using a resistive strain gauge sensor in a Wheatstone bridge configuration, (see figure 6). The Wheatstone bridge consists of four resistors $R_1 = R_2 = R_3 = R_4 = 600\,\Omega$. The strain gauge R_1 is placed on the micro bar and its resistance varies with strain. The micro bar is made out of silicon and has an initial length of $L = 20\,\mu\text{m}$ (with no force applied). The strain gauge is made of doped silicon and has a gauge factor of $\text{GF} = 30$.

A constant voltage,$V_{cc} = 4.000\,\text{V}$, is applied to the bridge. The voltage measured on the output of the Wheatstone bridge before and after applying the force is $V_{out} = 0.000\,\text{V}$ and $V_{out} = 0.005\,\text{V}$ respectively.

Calculate the length of the cantilever when the force is acting.

hint:

$$V_{out} = \left(\frac{R_3}{R_3 + R_4} - \frac{R_2}{R1 + R_2} \right) \cdot V_{cc}$$

The weight of the microbar is very small compared to the force P and the strain gauge is very thin compared to the microbar.

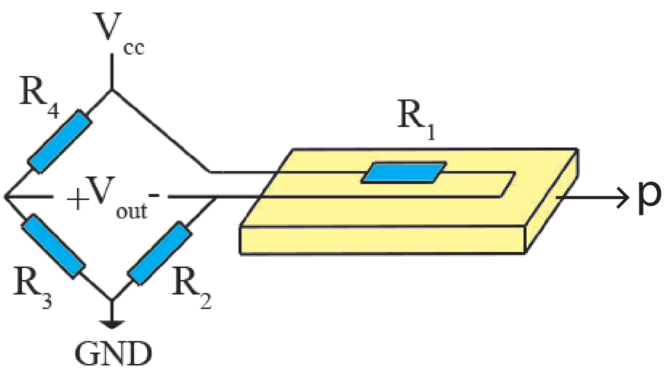

Figure 6: Microbar with strain gauge sensor to measure elongation.