Exercise 1

We consider a loaded beam with a cross-section in T-shape (fig. 1). The beam is characterized by the following values: $E = 20\,\text{GPa}$, $\nu = 0.2$, $t_1 = 2\,\text{cm}$, $t_2 = 1\,\text{cm}$, $\omega_1 = 1\,\text{cm}$, $\omega_2 = 4\,\text{cm}$, $L = 21\,\text{cm}$ and $F = 100\,\text{N}$. We define $z = 0$ at the top of the beam.

a) We will want to find the internal moment of the beam $M(x)$. Which axis should you consider for the calculation of the moment of area?

b) What is the distance z_c between the centroid and the top of the bar ? Calculate the moment of area at the centroid.

c) Determine the expression of $q(x)$. Deduce $V(x)$ and $M(x)$ by integration.

d) *Reminder: the formula for stress in a bent beam with a constant cross section is given by:* $\sigma_z(x) = \frac{M_y(x)}{I_y} \cdot (z - z_c)$*, where* z_c *is the position of the centroid.*
What is $\sigma_z(x)$ as a function of F and the geometrical dimensions ? Plot $\sigma_z(x)$ for the top of the beam $\sigma_{z=0cm}(x)$, the neutral axis $\sigma_{z=z_c}(x)$ and the bottom of the beam $\sigma_{z=3cm}(x)$.

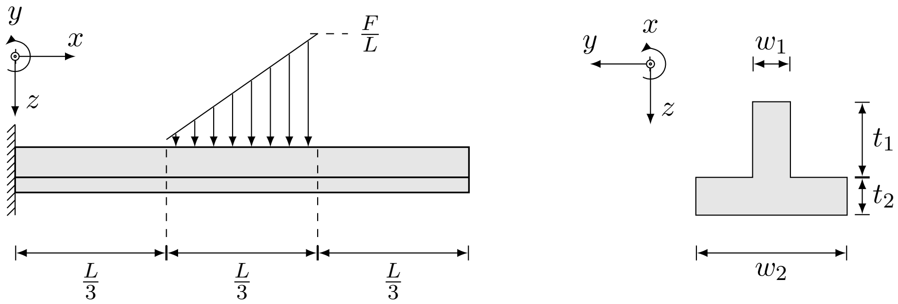

Figure 1: Schematic of the loaded bar and its cross-section.

Exercise 2

a) The cross-section of an H-beam is shown in fig. 2. Find the centroid and second moments of area I_y and I_z of the beam.

b) For the two cross-sections shown in fig. 3, find the second moments of area I_y and I_z.

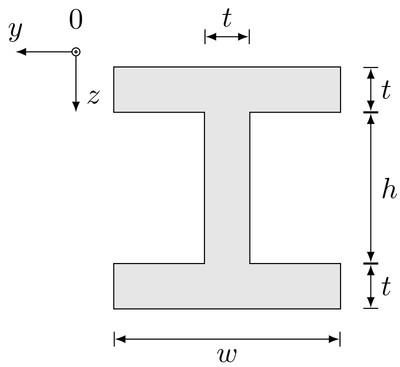

Figure 2: Cross-section of an H-beam.

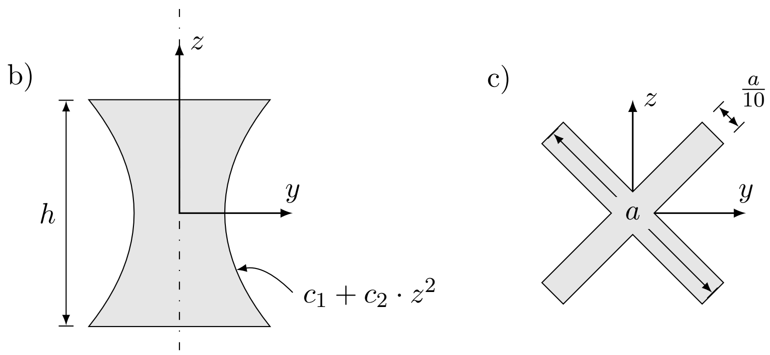

Figure 3: Cross-sections of more complicated beams.

Exercise 3

Occasionally, beams are made by joining two different materials, for example for a bimetal sensing element or actuator. Fig. 4 shows such a beam and its corresponding cross section.
Assume the top material is aluminium with a Young's modulus $E_{\mathrm{Al}} = 70\,\mathrm{GPa}$ and a thickness $t_1 = 10\,\mathrm{mm}$. The bottom material is copper with a Young's modulus $E_{\mathrm{Cu}} = 120\,\mathrm{GPa}$ and a thickness $t_2 = 7\,\mathrm{mm}$. The beam has a width $w = 20\,\mathrm{mm}$.

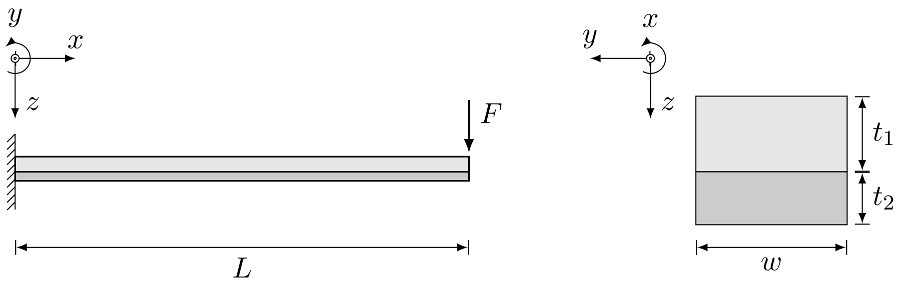

Figure 4: Composite beam under bending.

a) Find an expression for the neutral axis when the Young's modulus in z-direction is variable. Find the neutral axis of the given beam.

b) Find the equivalent flexural rigidity (EI) of the beam.