# Exercise 1

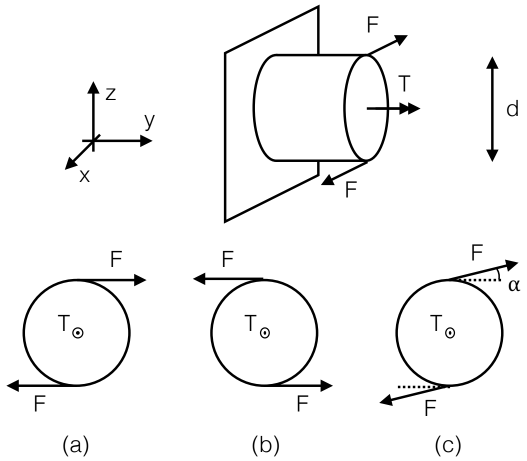


Figure 1: Simple cylindric bar under torsion, three different cases viewed from the right side.

A simple cylindric bar is clamped at one end and put under torsion by a force couple (fig. 1, forces F). As seen in class, these two forces can also be expressed as an applied torque (fig. 1, torque T).

For each of the three cases, express T as a function of F. Respect the sign conventions. What is special about case (c)?

*Reminder: the moment (or torque) of a force is defined according to fig. 2 and is given by:*

$$\overrightarrow{T} = \overrightarrow{r} \times \overrightarrow{F} \rightarrow T = rF_{\perp} = rF\sin\theta$$

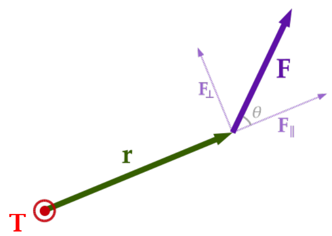


Figure 2: Definition of a torque.

## Exercise 2

A patient has had to amputate his leg and will receive a prosthesis. He used to play basketball and would like to continue to do so. The prosthesis should be built in order to support his movements. During one move, when he wants to pivot on one leg, the forces depicted in figure 3 will act on the prosthetic foot. To simplify the scenario, assume that the forces have their lines of action at a distance $b = 100\text{mm}$ from the outside of the tube, as shown in figure 3.

The allowable shear stress in the tube is 400MPa. The inner radius of the prosthetic tube is $r_1 = 10\text{mm}$ and the outer radius is $r_2 = 15\text{mm}$.

Calculate the maximum force that the patient can apply as depicted in figure 3, before the prosthesis breaks.

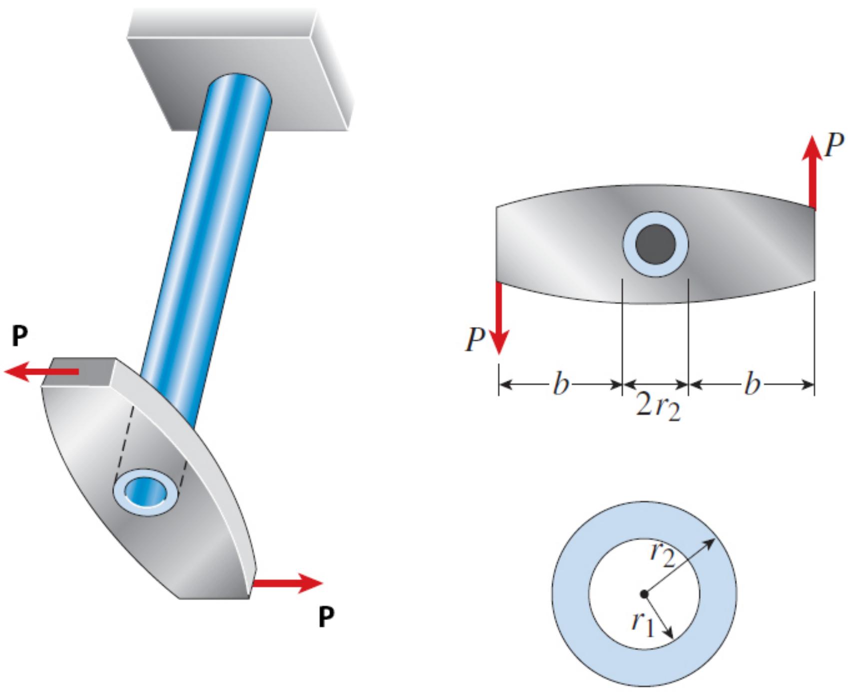


Figure 3: A very simplified foot prosthesis.

## Exercise 3

A tapered bar $AB$ of solid circular cross section is twisted by torques $T$ (see fig. 4). The diameter of the bar varies linearly from $d_A$ at the left-hand end to

$d_B$ at the right-hand end. The length of the bar is $L$ and the shear modulus of the bar material is $G$.

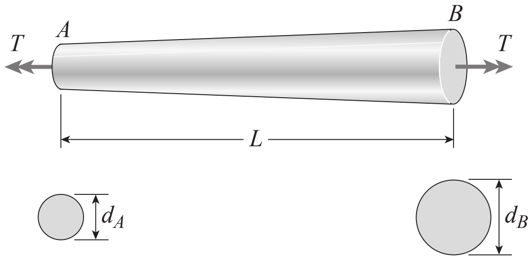


Figure 4: A tapered bar of solid circular cross section.

a) Derive the formula for the angle of twist of the tapered bar $AB$.

b) We know that a slightly different solid circular bar of diameter $d_A$ and length $L$, made of the same material and under the same torques $T$, has an angle of twist $\varphi = 0.1\,\text{rad}$. Knowing also that $\frac{d_B}{d_A} = \beta = 1.5$, calculate the angle of twist of the tapered bar.

## Exercise 4

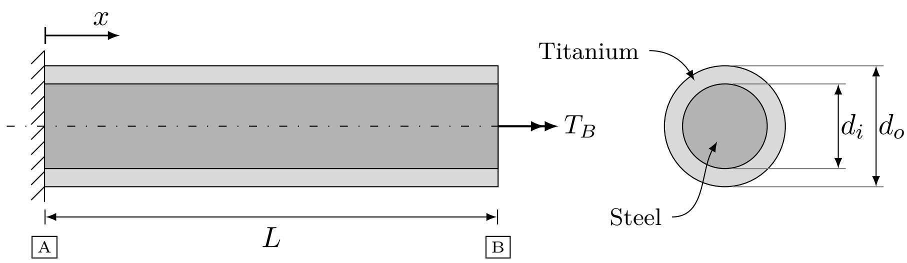


Figure 5: Composite bar made of an steel core with a titanium cladding under torsion.

A steel bar has been made biocompatible by cladding it with a hull of titanium (fig. 5). The part is then subjected to a torque $T_B = 20\,\text{N m}$. The steel core has a diameter of 8 mm, the cladding has an outer diameter of 10 mm and the whole bar has a length of 100 mm. Steel has a shear modulus $G_{\text{Steel}}$ of 79 GPa and titanium has a shear modulus $G_{\text{Ti}}$ of 41 GPa.

a) Find the torsional stiffness (torsional spring constant) of the beam, as well as the twist in point B.

b) Calculate the minimal and maximal shear stresses in the steel and the titanium part.

c) Draw the radial shear stress distribution $\tau(r)$ over the whole part.