Exercise 1

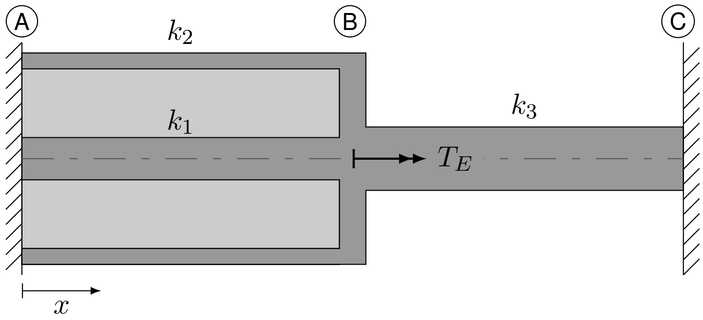


Figure 1: Cross section through a bar and tube system under torsion.

The torsion bar system in figure 1 consists of a hollow tube and two different bars connected in the middle with a stiff plate. The system is being twisted in the middle with an external torque T_E.

a) Find the twist at point B and the reaction torques at the walls A and C with the displacement stiffness method.

b) Calculate the reaction torque of the wall against the tube for $T_E = 1\,\mathrm{N\,m}$, $G = 10\,\mathrm{MPa}$ and lengths $L_{AB} = L_{BC} = 1\,\mathrm{m}$. Assume axially symmetric cross sections with a diameter $d_1 = 3\,\mathrm{cm}$ for the bar inside the tube, an outer tube diameter of $d_{2,o} = 10\,\mathrm{cm}$ and a tube wall thickness of $t_2 = 5\,\mathrm{mm}$, as well as a diameter $d_3 = 5\,\mathrm{cm}$ for the bar on the right.

Exercise 2

Two torques are applied to the structure defined figure 2. We want to determine the torques applied to the structure by the wall as well as the angles of twist in the structure. The structure is made of a homogeneous material with a modulus of rigidity G.

a) Define the nodes and elements. Reminder : each node corresponds to a unique torque and a unique angle.

b) Write the local stiffness matrices as a function of $k = \frac{\pi R^4 G}{L}$.

c) Assemble the local stiffness matrices into the global stiffness equation in which you will also incorporate the boundary conditions.

d) Solve the global stiffness equation and express the unknown variables as a function of k, T_A, T_B.

e) Plot the angle of twist of the tube and the cylinder along the x axis if $T_A = T_B = T$.

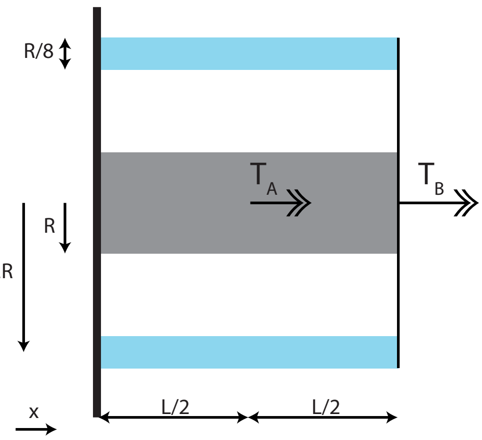


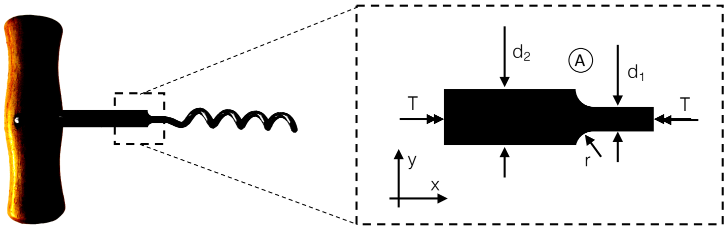
Figure 2: Cross section through a bar and tube system under torsion.

Exercise 3

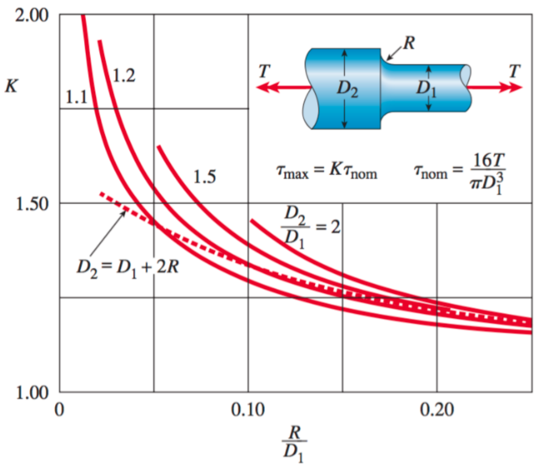
You have just learned that you got a great mark on your Structural Mechanics exam. Your friend Santiago however has a red face and upon asking, you find out that he failed miserably. Since you both have ample reason to drink, you decide to open a bottle of wine.

a) In his frustration, Santiago puts too much torque on his vintage corkscrew and it breaks at point A (fig. 3a). Calculate the maximum torque that he should not have exceeded. Use $d_1 = 1\text{mm}$, $d_2 = 1.5\text{mm}$, $r = 0.05\text{mm}$. The ultimate shear strength of the material is $\tau_U = 200\text{N/mm}^2$.

b) You take the bottle and use your patented Swiss army knife. Without surprise, you have no trouble opening the bottle. Knowing that driving a corkscrew into a cork results in a maximal torque of 0.1 Nm, calculate the safety factor that the manufacturer used on the army knife. The dimensions are $d_1 = 1.5\text{mm}$, $d_2 = 3\text{mm}$, $r = 0.35\text{mm}$. The material is steel, with $\tau_U = 360\text{N/mm}^2$.



(a) Illustration of the corkscrew: problem definition.



(b) Torsional stress concentration factor as a function of the geometry.

Figure 3: Illustration of the corkscrew and stress concentration factor graph.

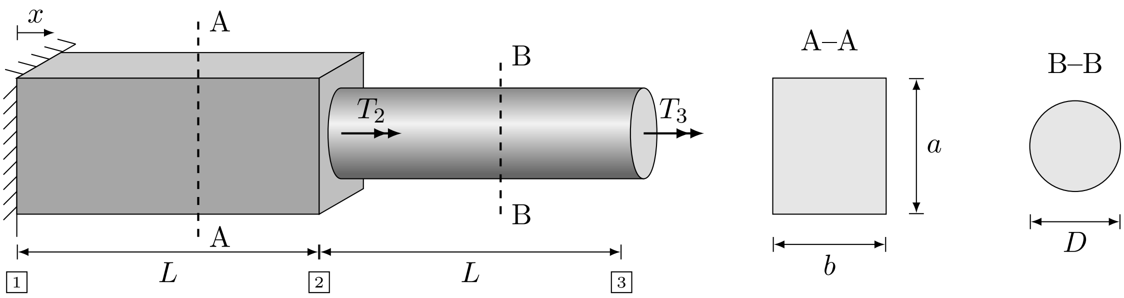


Figure 4: Bar system under torsion and associated cross-sections

Exercise 4

When bars under torsion have a non–circular cross–section, one can use correction factors to find the maximum shear stress $\tau_{\max}$ and the twist φ in those beams. The bar system under torsion shown in figure 4 consists of one bar with rectangular cross-section that is fixed to a wall, as well as a round bar attached to the rectangular bar. Both bars are made out of aluminium (Young's modulus $E = 70\,\mathrm{GPa}$, Poisson ratio $\nu = 0.35$). The bars have a length of $L = 40\,\mathrm{cm}$ each. The other dimensions are $a = 2.4\,\mathrm{cm}$, $b = 2\,\mathrm{cm}$ and $D = 1.8\,\mathrm{cm}$. The torques applied are $T_2 = 1.0\,\mathrm{N\,m}$ and $T_3 = 2.0\,\mathrm{N\,m}$.

a) What is the internal torque in the cylinder? And in the rectangular bar?

b) Find the maximum shear stress in the whole system.

c) What is the value of the shear modulus G for aluminium ? Find the angle of twist in point 2 and in point 3.