MATH-656
Numerical linear algebra for Koopman and Dynamic Mode Decomposition (DMD)

March 18, 2024

Abstract

General remarks

The projects assignments contain various versions of the DMD and its applications. Together with the material covered in the classes, they should contribute to your operative knowledge of the DMD and the Koopman operator frameworks.

Each project assignment defines the main topic and provides few references that should be used as starting points to explore the theme. The themes are selected to cover the aspects that are related to the interest of the group (randomized approach, tensor algorithms, sparse solutions, control theory). If you have a particular topic that you want to explore in the context of this course, you can make a proposal and we will define a project assignment together.

It is expected that you work in teams of two – choose one of the projects, work on the problem together, but submit two separate/individual reports. Also, you should prepare (as a team) a 15 minutes long presentation so that the rest of the class can learn about the theme and the results of your project. The teams work independently. Other details will be discussed in the class.

Contents

1 P1: Randomized DMD with applications to video processing

The goals of this project are:

- To learn about a particular real world application of the DMD: recorded video as a sequence of frames is considered a discrete dynamics and the elements of the DMD (its spatio-temporal representation of the data snapshots/frames) are interpreted in this context. This is then used for algorithmic separation of the foreground from the static background.
- To study an implementation of the DMD in detail. Video data can be high dimensional and randomized algorithms are good candidates for computational efficiency and accuracy that is sufficient for this application.
- To practice software development – data manipulation, implementation, testing and tuning a computational method, presentation and interpretation of the results.
- To practice technical writing and presenting.

The expected outputs of the project are:

- A technical report that contains detailed description of the problem, methods, implementation details and test examples. It has to be structured as a research paper. It is submitted as a PDF file.
- A software implementation that illustrates the performances on the data sets selected from the literature. The software is submitted in a zipped archive that contains, in separate folders, source codes, data, examples, documentation. (Large data files should be made available for download.) If you decide to post it on GitHub, a link to the repository will suffice. Matlab is preferred, and you can also use Python. A live presentation of the software might be requested.
- A short presentation (15 minutes talk) to the class.

In the references below, you can find descriptions of the algorithms, including links to the data sets. It is expected that you explore further references on this topic. It is not acceptable to merely reproduce one particular paper. Original ideas will be rewarded with extra credit.

References:

1. N. B. Erichson, S. L. Brunton, J. N. Kutz: Compressed Dynamic Mode Decomposition for Background Modeling, arXiv:1512.04205.
2. N. B. Erichson, C. Donovan: Randomized low-rank Dynamic Mode Decomposition for motion detection, Computer Vision and Image Understanding 146 (2016) 40–50.

3. J. Grosek and J. N. Kutz: Dynamic Mode Decomposition for Real-Time Background/Foreground Separation in Video, arXiv:1404.7592.

4. J. N. Kutz, X. Fu, S. L. Brunton and N. B. Erichson: Multi-resolution dynamic mode decomposition for foreground/background separation and object tracking, 2015 IEEE International Conference on Computer Vision Workshop (ICCVW), pp. 921-929, 2015.

5. I. Ul Haq, K. Fujii, Y. Kawahara: Dynamic mode decomposition via dictionary learning for foreground modeling in videos, Computer Vision and Image Understanding 199 (2020) 103022.

2 P2: The DMD with tensor data

In many applications, the numerical simulation data are naturally structured as multidimensional arrays, i.e. tensors. By the standard matricization and vectorization operations, the natural two- and three-dimensional data are organized and processed as columns of the data matrix. In the case of high dimensional data, this approach becomes difficult because of high data volume and computational complexity.

If your current research/interest or the topic of your thesis is in the area of numerical methods for tensors, then this research project is an opportunity to apply the techniques of low rank tensor approximation/representation to the DMD. The idea is to use compressed tensor formats and formulate the DMD method in the computational framework of tensors.

The goals of this project are:

- To study an implementation of the DMD for data given in some tensor format. If tensor methods are in the focus of your research/thesis then this is an opportunity to apply your expertise in an important application.
- To practice software development – data manipulation, implementation, testing and tuning a computational method, presentation and interpretation of the results.
- To practice technical writing and presenting.

The expected outputs of the project are:

- A technical report that contains detailed description of the problem, methods, implementation details and test examples. It has to be structured as a research paper. It is submitted as a PDF file.
- A software implementation that illustrates the performances on the data sets selected from the literature. The software is submitted in a zipped archive that contains, in separate folders, source codes, data, examples, documentation. (Large data files should be made available for download.) If you decide to post it on GitHub, a link to the repository will suffice. Matlab is preferred, and you can also use Python. A live presentation of the software might be requested.
- A short presentation (15 minutes talk) to the class.

In the references below, you can find descriptions of the algorithms, including links to the data sets. It is expected that you explore further references on this topic. It is not acceptable to merely reproduce one particular paper. Original ideas will be rewarded with extra credit.

References:

1. S. Klus, P. Gelß, S. Peitz, Ch. Schütte: Tensor-based dynamic mode decomposition, Nonlinearity, 31(7), 2018.

2. S. Klus, Ch. Schütte: Towards tensor-based methods for the numerical approximation of the Perron–Frobenius and Koopman operator, Journal of Computational Dynamics, 2016, 3(2): 139-161

3. K. Li, S. Utyuzhnikov: Tensor train–based higher–order Dynamic Mode Decomposition for dynamical systems, Mathematics 2023, 11(8), 1809.

4. https://cgl.ethz.ch/research/visualization/data.php

5. http://dmdbook.com/

3 P3: DMD in control

DMD can be applied for Model Predictive Control. Those of you with the control theory background can use this project to add the DMD based MPC technology to your repertoire.

The goals of this project are:

- To master application of the DMD in a Model Predictive Control framework. If your research/thesis includes MPC, then this is an opportunity to test DMD approach using some of your test examples.
- To practice software development – data manipulation, implementation, testing and tuning a computational method, presentation and interpretation of the results.
- To practice technical writing and presenting.

The expected outputs of the project are:

- A technical report that contains detailed description of the problem, methods, implementation details and test examples. It has to be structured as a research paper. It is submitted as a PDF file.
- A software implementation that illustrates the performances on the data sets selected from the literature. The software is submitted in a zipped archive that contains, in separate folders, source codes, data, examples, documentation. (Large data files should be made available for download.) If you decide to post it on GitHub, a link to the repository will suffice. Matlab is preferred, and you can also use Python. A live presentation of the software might be requested.
- A short presentation (15 minutes talk) to the class.

In the references below, you can find descriptions of the algorithms, including links to the data sets. It is expected that you explore further references on this topic. It is not acceptable to merely reproduce one particular paper. Original ideas will be rewarded with extra credit.

References:

1. H. Arbabi, M. Korda, I. Mezić: A data-driven Koopman model predictive control framework for nonlinear flows, arXiv:1804.05291.
2. J. L. Proctor, S. L. Brunton, J. N. Kutz: Dynamic Mode Decomposition with Control, SIAM Journal on Applied Dynamical Systems 15 (1), 2016. pp. 142-161.
3. `https://github.com/arbabiha/KoopmanMPC_for_flowcontrol`

4 P4: DMD with noisy sensor data

In data driven application, the sensor data are corrupted by noise. To enhance robustness to noise, various modifications of the DMD have been proposed.

The goals of this project are:

- To learn on data acquisition and methods of mitigating the problem of noise in the data. One possible techniques is to use the total least squares to define the DMD matrix. Another is the so called forward-backward DMD.
- To practice software development – data manipulation, implementation, testing and tuning a computational method, presentation and interpretation of the results.
- To practice technical writing and presenting.

The expected outputs of the project are:

- A technical report that contains detailed description of the problem, methods, implementation details and test examples. It has to be structured as a research paper. It is submitted as a PDF file.
- A software implementation that illustrates the performances on the data sets selected from the literature. The software is submitted in a zipped archive that contains, in separate folders, source codes, data, examples, documentation. (Large data files should be made available for download.) If you decide to post it on GitHub, a link to the repository will suffice. Matlab is preferred, and you can also use Python. A live presentation of the software might be requested.
- A short presentation (15 minutes talk) to the class.

In the references below, you can find descriptions of the algorithms, including links to the data sets. It is expected that you explore further references on this topic. It is not acceptable to merely reproduce one particular paper. Original ideas will be rewarded with extra credit.

References:

1. S. T. M. Dawson, M. S. Hemati, M. O. Williams, C. W. Rowley: Characterizing and correcting for the effect of sensor noise in the dynamic mode decomposition, Experiments in Fluids, Vol. 57, No. 3, 2016, article 42.
2. M. S. Hemati, C. W. Rowley, E. A. Deem, L. N. Cattafesta: De-biasing the Dynamic Mode Decomposition for Applied Koopman Spectral Analysis of Noisy Datasets, Theoretical and Computational Fluid Dynamics, Vol. 31, No. 4, 2017, pp. 349-368.

3. M. Y. Ali, A. Pandey and J. W. Gregory: Dynamic Mode Decomposition of Fast Pressure Sensitive Paint Data, Sensors 2016, 16, 862. (This reference included just as an example of a method for data acquisition.)

5 P5: Sparse DMD

The DMD is used to reveal latent structures in the dynamics. As we discussed in the class, this is accomplished by representing the data snapshots in terms of the modes – the eigenvectors of the DMD matrix that have been identified in the DMD algorithm. A method of finding small number of most relevant modes is based on the structured least squares solution of the reconstruction problem (as discussed in the class) but with an additional constraint that enforces/favors sparse solution, i.e. the reconstruction error should be small, but with small number of nonzero reconstruction coefficients.

The goals of this project are:

- To learn on the DMD snapshot reconstruction using sparse least squares solution.
- To practice software development – data manipulation, implementation, testing and tuning a computational method, presentation and interpretation of the results.
- To practice technical writing and presenting.

The expected outputs of the project are:

- A technical report that contains detailed description of the problem, methods, implementation details and test examples. It has to be structured as a research paper. It is submitted as a PDF file.
- A software implementation that illustrates the performances on the data sets selected from the literature. The software is submitted in a zipped archive that contains, in separate folders, source codes, data, examples, documentation. (Large data files should be made available for download.) If you decide to post it on GitHub, a link to the repository will suffice. Matlab is preferred, and you can also use Python. A live presentation of the software might be requested.
- A short presentation (15 minutes talk) to the class.

In the references below, you can find descriptions of the algorithms, including links to the data sets. It is expected that you explore further references on this topic. It is not acceptable to merely reproduce one particular paper. Original ideas will be rewarded with extra credit.

References:

1. M. R. Jovanović, P. J. Schmid, and J. W. Nichols: Sparsity-Promoting Dynamic Mode Decomposition" Phys. Fluids, vol. 26, no. 2, p. 024103 (22 pages), 2014.

2. M. R. Jovanović, P. J. Schmid A N D J. W. Nichols: Low-rank and sparse dynamic mode decomposition, Stanford Center for Turbulence Research, Annual Research Briefs 2012.

3. http://people.ece.umn.edu/~mihailo/software/dmdsp/index.html