INTRO TO DYNAMICAL SYSTEMS FALL 2024, PROBLEM SET 9

(1) Prove Lemma 3.2 in Lecture 9.pdf
(2) Give a careful proof of Lemma 3.3 in Lecture 9.pdf.
(3) Using the result from the second exercise in problem set 8, prove that if $0 \in \mathbb{R}^n$ is a hyperbolic fixed point of the system of ODEs

$$\dot{y}_j(t) = f_j(y_1, y_2, \ldots, y_n),\ j = 1, 2, \ldots, n, \tag{1}$$

and more specifically $Df\big(0\big) = \left(\frac{\partial f_j}{\partial y_i}\right)_{1 \leq i,j \leq n}$ *only has eigenvalues with negative real part*, then there is a neighbourhood

$$0 \in U \subset \mathbb{R}^n$$

such that the solution of (1) with initial condition $\underline{y}(0) \in U$ satisfies a bound of the form

$$\big|\underline{y}(t)\big| \leq C \cdot e^{-ct},\ t \geq 0$$

for suitable constants $C, c > 0$.