INTRO TO DYNAMICAL SYSTEMS FALL 2024, PROBLEM SET 7

(1) By considering the map $T : S^1 \longrightarrow S^1$ given by $Tz = z^{10}$, show that for almost every $x \in (0, 1]$ it holds that writing

$$x = 0.x_1x_2\dots$$

in decimal expansion, we have that

$$\lim_{N\to\infty} \frac{1}{N} \cdot \big|\{j \in [1, 2, \dots, N],\ \ x_j = 0\}\big| = \frac{1}{10}.$$

(2) Complete the proof of Lemma 1.2 in Lecture8.pdf, i. e. determine the constants $c, \theta_\pm$ there.

(3) Let $A \in \mathrm{Mat}\big(n \times n; \mathbb{R}\big)$ be a hyperbolic matrix, and let

$$\phi(x) = Ax + \hat{\phi}(x),\ \psi(x) = Ax,$$

where $\hat{\phi}$ is in $C^1_b\big(\mathbb{R}^n, \mathbb{R}^n\big)$, i. e, bounded and once continuously differentiable. Further assume that the Lipschitz constant for $\hat{\phi}$ is sufficiently small, so that the conditions of Prop.4.2 of Lecture8.pdf are satisfied. Then show that if $\phi(0) = 0$ and[1]

$$\mathrm{trace}\big(D\hat{\phi}(0)\big) \neq 0,$$

the homeomorphism h whose existence is asserted in Prop. 4.2 cannot be a C^1-diffeomorphism.

[1]Here $D\hat{\phi}$ is the Jacobian of the map