INTRO TO DYNAMICAL SYSTEMS FALL 2024, PROBLEM SET 3

(1) Show that the solution $u(x) = x + \hat{u}(x)$ given in Theorem 2.1 of Lecture3.pdf satisfies the bound
$$\left\|\hat{u}\right\|_{L^\infty} \leq \left\|\hat{\psi}\right\|_{L^\infty}.$$

(2) Show that if we equip $\mathbb{R}$ with the *probability measure* $\pi^{-1}\frac{dx}{1+x^2}$, then the map $T : \mathbb{R} \longrightarrow \mathbb{R}$ given by
$$T(x) = \frac{1}{2}\big(x - \frac{1}{x}\big),\ x \neq 0,\ T(0) = 0$$
is measure preserving.

(3) Let X be a probability space (i. e. $m(X) = 1$) and let $T : X \longrightarrow X$ measure preserving. Then if $A \subset X$ is measurable with $m(A) > 0$, there is an integer $n \leq m(A)^{-1}$ and such that
$$m\big(A \cap T^{-n}(A)\big) > 0.$$

(4) Let $T : z \to z^3$ the 'tripling map' of the circle. Show that there is a measure zero non-countable set $K \subset S^1$ which is mapped into itself by T.*Hint: Cantor set.*