

**INTRO TO DYNAMICAL SYSTEMS FALL 2024,
PROBLEM SET 2**

- (1) Let T be a continuous map on a metric space X without isolated points. Show that if x has dense orbit under T , then so does $T^n x$ for every $n \geq 1$.
- (2) Suppose that X is a metric space with at least one isolated point. Then if $T : X \rightarrow X$ is continuous and such that for any non-empty open sets U, V there is $n \geq 0$ such that $T^n(U) \cap V \neq \emptyset$, then X is a finite set and $X = \mathcal{O}^+(x)$.
- (3) By considering the set of 2^n -th roots of unity, $n \geq 1$, show that the second part of the Birkhoff transitivity theorem is violated if we drop the extra assumptions on (X, d) .