

**INTRO TO DYNAMICAL SYSTEMS FALL 2024,
PROBLEM SET 1**

- (1) Carefully show that the map

$$\pi : \mathbb{R}^n \longrightarrow \mathbb{T}^n,$$

where $\mathbb{T}^n = S^1 \times S^1 \times \dots \times S^1$ is the n -dimensional torus, given by

$$\pi(\mathbf{x}) = (e^{2\pi i x_1}, e^{2\pi i x_2}, \dots, e^{2\pi i x_n}), \mathbf{x} = (x_1, \dots, x_n)$$

is a covering map.

- (2) Show that Theorem 4.4 from lecture1.pdf is wrong in general if we only assume f to be Lebesgue integrable.
- (3) Give a detailed proof of Theorem 4.7 in lecture1.pdf.
- (4) Show that the condition $\sum_{k=1}^n \alpha_k \cdot j_k \notin \mathbb{Z}$ for $j_k \in \mathbb{Z}$ is satisfied for $n = 3$, $\alpha_1 = \sqrt{2}$, $\alpha_2 = \sqrt{3}$, $\alpha_3 = \sqrt{5}$.