

**INTRO TO DYNAMICAL SYSTEMS FALL 2024,
PROBLEM SET 11**

- (1) Show that the set of non-diophantine irrational numbers in $\mathbb{R} \setminus \mathbb{Q}$ is of Lebesgue measure 0.
- (2) Recall (or look up) the three lines lemma of Hadamard. Using it, show the inequality

$$\sum_{k=0}^{\infty} k^d \cdot |\zeta|^k \leq C(d) \cdot (1 - |\zeta|)^{-d-1}$$

provided $|\zeta| < 1$, and $d \geq 0$. Can you estimate $C(d)$?

- (3) Given any non-polynomial holomorphic function $u(z)$ on a disc $B_r(0)$, $r > 0$, show that there is an irrational number $\alpha \in \mathbb{R}$ such that

$$w(\lambda z) - \lambda w(z) = u(z), \quad \lambda = e^{2\pi i \alpha},$$

does not have a holomorphic solution on any disc $B_\delta(0)$.