1.
2. Let $z \in K$ and $n \in \mathbb{N}$ such that $z^n = 0$, i.e., z is a nilpotent element in K. Note that for any $\xi \in K$ we have $(z\xi)^n = 0$ since K is commutative. Since $[\times(z\xi)^n] = [\times z\xi]^n$, we know that 0 is the only eigenvalue of $[\times z\xi]$. In particular, for all $\xi \in K$, we have
$$\langle z, \xi\rangle_{K/Q} = \mathrm{tr}[\times z\xi]_{K/Q} = 0.$$
Let $z, \xi \in K$ nilpotent and let $m, n \in \mathbb{N}$ such that $\xi^m = z^n = 0$. Let $\alpha \in k$. Then
$$(\alpha\xi + z)^{m+n} = \sum_{k=0}^{m+n} \binom{m+n}{k} \alpha^k \xi^k z^{m+n-k}$$
and, since $m + n - k \geqslant n$ unless $k > m$, it follows that $\alpha\xi + z$ is nilpotent. In particular, the set Nil_K of nilpotent elements is a subspace and by the above argument it is a subspace of $\ker\Phi$, where $\Phi\colon K \to K^*$ is the k-linear map given by
$$\forall v, z \in K \quad \Phi(z)(v) = \langle z, v\rangle_{K/k}.$$
Since K is non-reduced, $\mathrm{Nil}_K \neq \{0\}$ and, hence, Φ has non-trivial kernel.
For what follows, let $r = \dim\ker\Phi$ and fix a decomposition $K = U \oplus \ker\Phi$ with bases $\mathcal{B}_U = (u_1, \ldots, u_{d-r})$ and $\mathcal{B}_0 = (w_1, \ldots, w_r)$ of U and $\ker\Phi$ respectively. The tuple $\mathcal{B} = (v_1, \ldots, v_d) \in K^d$ given by
$$v_i = \begin{cases} w_i & \text{if } 1 \leqslant i \leqslant r, \\ u_{i-r} & \text{otherwise,} \end{cases}$$
is a k-basis of K. In particular, for any tuple $\boldsymbol{\xi} \in K^d$ there exists a matrix $A = (a_{ij})_{1\leqslant i,j\leqslant d} \in k^{d\times d}$ such that
$$\forall 1 \leqslant i \leqslant d \quad \xi_i = \sum_{k=1}^{d} a_{ik} v_k,$$
hence k-linearity of the trace implies that
$$\mathrm{disc}_{K/k}(\boldsymbol{\xi}) = \det(A)^2 \,\mathrm{disc}_{K/k}(\mathcal{B}) = \det(A)^2 \det\left(\langle v_i, v_j\rangle_{K/k}\right) = 0,$$
since $\langle v_i, v_j\rangle_{K/k} = 0$ whenever $\min\{i, j\} \leqslant r$.
3.