Optimal transport, Fall semester
EPFL, Mathematics section, Dr. Xavier Fernández-Real
Exam guidelines

The exam will consist of two parts:

- First part: **25 minutes**. The student will be given a topic from the list of topics. During this time the student should prepare an oral presentation on the assigned topic to be exposed in the following part.
- Second part: **25 minutes**. Oral presentation. During this time, the student will present the (pre-assigned) topic prepared during the previous part. The examiners will pose questions and ask for further clarifications from the student. This part is expected to be interactive between the student and the examiners.

The list of topics for the exam is the following:

(i) Kantorovich duality

(ii) Brenier's theorem

(iii) Wasserstein distance

(iv) Displacement convexity

(v) The heat equation and the JKO scheme.

Some remarks:

- The student is given some freedom regarding what they want to prepare about a topic, with the understanding that any missing information can be asked as a follow-up or side question by the examiners.
- It is expected that the student prepares the general theory (with proofs) of the corresponding topic, with sufficient depth to be explained in about 20 minutes.
- The student should:
  - **select the definitions, proofs, examples**, as well as the level of detail, that constitute the main focus of the presentation;
  - clearly **state the main theorem**;
  - **define** the main objects;
  - **provide simple examples** during the exposition.

Example. A sample examination could be the following (of course this is just an example!).

Topic: Brenier's theorem.

The student states Brenier's theorem, and follows the proof (for example, as in the book of the course, Steps 1–4). The proof does not need to be extremely detailed, an idea of each step is enough, and the examiners might ask for further detail in a specific step. For example, the proof starts with the existence of an optimal transport plan, which can be stated like that during the proof.

During the exposition, the student states (without proof) that optimal plans exist and have support contained in the sub-differential of a convex function, and the examiners might ask to briefly explain why the existence of an optimal plan holds or why do we get precisely "convex functions" at this step. This may involve mentioning the Rockafellar theorem and the c-cyclical monotonicity.

The student finishes the proof as expected, at which point the examiners can ask for further clarifications. Some sample questions may be: where was the absolute continuity of the original measure used? How is the statement modified for general costs, and what would be the main differences from the proof presented? Can the student show the existence of an inverse map, and under which assumptions? How, later in the course, is Brenier's optimal transport map involved in the construction of a Wasserstein geodesic between two measures?

When preparing the exam, you are encouraged to ask yourself (or to other students in small groups) 3/4 questions for each theme, as in the example above.

It is not expected that the students answer all questions with full details in order to get a 6.