09 Finite Differences 2

It seems that FTBS works most reasonable.
What about Burgers' equation?

$$\partial_t u + \partial_x (u^2) = 0$$

$$\partial_t u + 2u\, \partial_x u = 0$$

Both forms are mathematically equivalent but inspire different finite-difference methods.

a)
$$\frac{1}{k}\left(U_j^{n+1} - U_j^n\right) + \frac{(U_j^n)^2 - (U_{j-1}^n)^2}{h} = 0$$

$$U_j^{n+1} = U_j^n - \frac{k}{h}\left((U_j^n)^2 - (U_{j-1}^n)^2 \right)$$

b)
$$\frac{1}{k}\left(U_j^{n+1} - U_j^n\right) + 2\, U_j^n \frac{U_j^n - U_{j-1}^n}{h} = 0$$

$$U_j^{n+1} = U_j^n - \frac{2k}{h} U_j^n \left(U_j^n - U_{j-1}^n \right)$$

How well do those work in practice?

$\Omega_x = [0,1]$, periodic BC

$u_0(x) = 1 + \frac{1}{5} \cdot \sin(2\pi x)$

The characteristics will go from left to right

a) is called "conservative" whereas b) is not.