02 Motivation

- We consider a quantity over a spatial interval
- Let $\rho(x,t)$ be the density of a substance at point x and time t
- The total amount over the interval $[x_1, x_2]$ is

$$M(t) = \int_{x_1}^{x_2} \rho(x,t) \, dx$$

Let $\nu(x,t)$ denote the velocity of the substance at (x,t), then physics gives us

$$\partial_t \int_{x_1}^{x_2} \rho(x,t) \, dx = \rho(x,t)\,\nu(x_1,t) - \rho(x,t)\,\nu(x_2,t)$$

$$= -\int_{x_1}^{x_2} \partial_x \big(\rho(x,t)\,\nu(x,t) \big) \, dx$$

Combining this gives

$$\int_{x_1}^{x_2} \partial_t \rho(x,t) \, dx = -\int_{x_1}^{x_2} \partial_x \big(\rho(x,t)\,\nu(x,t) \big) \, dx$$

This is valid for any time t and any subinterval $[x_1, x_2]$, hence we get pointwise

$$\partial_t \rho(x,t) = -\partial_x \big(\rho(x,t)\,\nu(x,t) \big)$$