Solutions for Statistical analysis of network data – Sheet 5

1. Consider the network with adjacency matrix

$$A_1 = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Please form the $n \times n$ symmetric matrix of all the distances between the nodes in this network. We note that

$$D = \begin{pmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 2 & 2 & 1 & 0 \end{pmatrix}$$

The two centralities are $\begin{pmatrix} 1 & 1 & 4/3 & 4/5 \end{pmatrix}$ and $\begin{pmatrix} 5/2 & 5/2 & 3 & 2 \end{pmatrix}$.

2. Consider the network with adjacency matrix

$$A_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

We note that

$$D = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{pmatrix}$$

The two centralities are $\begin{pmatrix} 2/3 & 1 & 1 & 2/3 \end{pmatrix}$ and $\begin{pmatrix} 1.8333 & 5/2 & 5/2 & 1.8333 \end{pmatrix}$. The efficiency is 0.7222 .

3. Consider the network with adjacency matrix

$$A_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

We note that

$$D = \begin{pmatrix} 0 & 2 & 2 & 2 & 1 \\ 2 & 0 & 2 & 2 & 1 \\ 2 & 2 & 0 & 2 & 1 \\ 2 & 2 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

The two centralities are $\begin{pmatrix} 0.5714 & 0.5714 & 0.5714 & 0.5714 & 1 \end{pmatrix}$ and $\begin{pmatrix} 5/2 & 5/2 & 5/2 & 5/2 & 4 \end{pmatrix}$. The efficiency is 0.7 .

4. We can calculate the triangles using the formula

$$X_{C_3} = \frac{1}{6} \text{ trace } A^3, \quad X_{P_3} = \frac{1}{2} \sum_i^n d_i \left(d_i - 1\right)$$

We note $X_{C_3}\left(G_1\right) = 1, X_{P_3}\left(G_1\right) = 5$ and finally $CC_{G_1} = 1/5$. For the second network $X_{C_3}\left(G_2\right) = 0$, $X_{P_S}\left(G_1\right) = 2$ and finally $CC_{G_1} = 0/2$. In the final case, $CC_{G_1} = 0$.

5. Already calculated above.