

## Exercises for Statistical analysis of network data – Sheet 7

1. Assume we observe two networks  $G_1$  and  $G_2$  with adjacency matrix  $A_1$  and  $A_2$ . These are given respectively by

$$A_1 = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}, \quad (1)$$

and

$$A_2 = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}. \quad (2)$$

Determine the Hamming and Jaccard distance between the two. What is the estimated edge density for these two graphs?

2. Assume instead we have two networks  $G_1$  and  $G_2$  with adjacency matrix  $A_1$  and  $A_2$ . These are given respectively by

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}, \quad (3)$$

and

$$A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}. \quad (4)$$

Determine the Hamming and Jaccard distance between the two. What is the estimated edge density for these two graphs?

3. Start from the exponential random graph formula of

$$\Pr\{\mathbf{A} = \mathbf{a}\} = \left(\frac{1}{\kappa}\right) \exp\left\{\sum_H \theta_H g_H(\mathbf{a})\right\},$$

with  $\kappa$  the normalizing constant. Assume that for any given pair of vertices, the presence or absence of an edge between that pair is independent of the status of possible edges between any other pair of vertices. That is  $A_{ij}$  is independent of  $A_{i'j'}$  for any non-equivalent doublets.

- Which configurations  $H$  are therefore eliminated in the expression?
- Simplify the expression under that assumption.
- Show how to recover the Erdos-Renyi model from this expression.