Solutions for Statistical analysis of network data – Sheet 2

1. Assume we draw observations from a configuration model with

$$\pi_i = \frac{\theta_n}{i^\gamma}, \quad i = 1, \ldots, n.$$

Calculate the degrees from the realized adjacency matrix A via

$$d_i = \sum_{j \neq i} A_{ij}.$$

   (a) Calculate the expectation of d_i. You may use the result that for large n (Gradshteyn + Ryzkin 0.233 , and an integral squeezing argument):

$$\sum_{j=1}^{n} \frac{1}{j^p} \approx \begin{cases} \zeta(p) & \text{if} \quad p > 1 \\ \log(n)(1 + o(1)) & \text{if} \quad p = 1 \\ \frac{n^{1-p}}{1-p}(1 + o(1)) & \text{if} \quad 0 < p < 1 \end{cases}.$$

   (b) Calculate the variance of d_i.

   (c) Calculate the dispersion of d_i. Are there instances when the dispersion approaches unity?

2. Show that the degree-corrected stochastic blockmodel is exchangeable.

3. Calculate the expected degree of node i from the random dot product graph.

4. Reformulate the stochastic block model as a random dot product graph. How do we select the latent dimension, call it d, of the RDGP relative to k the number of blocks.

5. What is the size of the automorphism group of an edge?

6. What is the size of the automorphism group of a triangle?