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# Counting subgraphs in random graphs

Let $F$ be a fixed deterministic unlabelled graph. Let $G$ be a random graph generated from random graph model $M$. Assume $|V(G)| = n \in \mathbb{N}^+$.

Define $X_F(G)$ the # of copies of $F$ in graph $G$. With $v = |V(F)|$, there are exactly $\binom{n}{v} v! / \mathrm{aut}(F)$ copies of $F$ in $K_n$ (Janson Luczak & Rucinski) (p 55).

Let us try calc again. To enable calculation, let $F$ have vertex set $\{1, \ldots, |V(F)|\}$ and $G$ $\{1, \ldots, n\}$. This is one (arbitrary) choice of labelling. As $G$ is ER, all choices are equivalent.

We now go through the calculation.

$$\mathbb{E}\{X_F(G)\} = \mathbb{E}\Big\{ \sum I(F \hat{=} F') \Big\}$$

$F \subseteq G$

this is "isomorphic to" not equal to see p 8 lecture 2. This counts for all relabellings.

$$= E\left\{ \sum_{F' \subseteq K_n} \mathbb{I}(F \equiv F')\, \mathbb{I}(F' \subseteq G) \right\}$$

fixed labelled graph

from sum

$$= \sum_{F' \subseteq K_n} \mathbb{I}(F \equiv F')\, \Pr(F' \subseteq G)$$

as the graph was "isomorphic" we don't need to allow all labellings

Note that

$$\Pr(F' \subseteq G) = p^{|e(F)|}$$

as $|e(F')| = |e(F)|$

$$E(X_F(G)) = \sum_{F' \subseteq K_n} \mathbb{I}(F \equiv F')\, p^{|e(F)|}$$

$$= p^{|e(F)|} \sum_{F' \subseteq K_n} \mathbb{I}(F \equiv F')$$

$$= p^{|e(F)|} \binom{n}{v} v! / \mathrm{aut}(F)$$

$\square$ QED