EXAMEN PROPEDEUTIQUE DE PROBABILITES AVANCEES

Probabilités avancées

15 Juin 2012

Length of Exam : 3h (from 15h to 18h)

Allowed : Nothing.

No calculating machines are permitted
Veuillez svp. commencer chaque exercice sur une nouvelle feuille.

Please write clearly your name

Name :______________________________ **First name :** ______________________________

Département : ____________________

Exercice	Points
1	
2	
3	
4	
5	
Total :	

1a) Let P and Q be two probabilites on the Borels of $\mathbb{R}^2$ satisfying $\forall\ x,\ y\ \in \mathbb{R}$,

$$P\left((-\infty, x] \times (-\infty, y]\right) \ = \ Q\left((-\infty, x] \times (-\infty, y]\right),$$

Dies it follow that the two probabilities sont are the same ?.
b) Give, with justification, an example of an algebra that is not a sigma algebra.

c) For $Q,\ P$ two probabilities on $(\Omega, \mathcal{F})$ so that for each $A \in \mathcal{F},\ Q(A) \ = \ \int_A V dP$ where V is positive and integrable (with respect to P. Show that if X_n converge in probability to X on $(\Omega, \mathcal{F}, P)$, then they also do so for $(\Omega, \mathcal{F}, Q)$.

2) Give a definition of the convergence in distribution, then add two equivalent conditions. Let $X_1, X_2, \cdots$ be i.i.d. random variables. having fonction characteristic function ϕ that satisfies

$$\phi(t) \ \sim \ 1 - \sqrt{|t|}$$

as $t \to 0$. Show carefully that

$$\frac{X_1 + X_2 + \cdots X_n}{n^2} \ \overset{D}{\to} W$$

(as n becomes large) for some variabe W.
Use this to deduce two properties of the limit distribution W ((with justification).

3) Let $X_1, X_2, \cdots$ be i.i.d. random variables which take integer values :

$$P(X_1 \ = \ (-1)^m m) \ = \ \frac{C}{m^2 log(m)}$$

where $C \ = \ \sum_{m \geq 1} \frac{1}{m^2 m log(m)}$. Does $E[X_1]$, exist ? Show that there is $\mu \in (-\infty, \infty)$ tel que

$$\frac{X_1 + X_2 + \cdots X_n}{n} \ \overset{pr}{\to} \mu$$

Does $\liminf_{n \to \infty} \frac{X_1 + X_2 + \cdots X_n}{n} \ = \ \mu$? Does the 'event $X_n = n$ hsppen for infinitely many n ?

4)Show carefully that the sigma algbr̀e generated by the Lipschitzien functions on $\mathbb{R}^2$ is the same as the Borellien subsets of $\mathbb{R}^2$.
For X_n, Y_n the independant r.v.s with $X_n \sim U(0, n^2)$ and $Y_n \sim \mathcal{E}\ xp(\frac{1}{n})$, does there exist infinitely many n with $Y_n > X_n$? Does $\frac{X_n}{n^2}$ converge in probability to $\frac{1}{2}$?

5)Give an example of a sequence of random variables which converges in L_1 not a.s..
b) The function positive, integrable sur $\mathbb{R}_+, f$ satisfies

$$\forall t \ \int_0^\infty cos(tx) f(x) dx \ = \ e^{-t^2}.$$

What is $\int_0^1 f(x) dx$.