

Solutions de la série n°11

Solution de l'exercice 1 :

$$1. \frac{\frac{\overline{\varphi \vdash \varphi}^{ax}}{\varphi, \psi \vdash \varphi}^{aff} \rightarrow_i}{\varphi \vdash \psi \rightarrow \varphi} \rightarrow_i \quad 2. \frac{\frac{\overline{\varphi \vdash \varphi}^{ax}}{\varphi \vdash \varphi \vee \psi}^{vig} \rightarrow_i}{\vdash \varphi \rightarrow (\varphi \vee \psi)} \rightarrow_i \quad 3. \frac{\frac{\overline{\varphi \wedge \psi \vdash \varphi \wedge \psi}^{ax}}{\varphi \wedge \psi \vdash \varphi}^{\wedge eg} \rightarrow_i}{\vdash (\varphi \wedge \psi) \rightarrow \varphi} \rightarrow_i$$

$$4. \frac{\frac{\frac{\overline{\neg \varphi \vdash \neg \varphi}^{ax}}{\neg \neg \varphi, \neg \varphi \vdash \perp} \perp_c}{\neg \neg \varphi \vdash \varphi} \rightarrow_i}{\vdash \neg \neg \varphi \rightarrow \varphi} \rightarrow_i \quad \frac{\frac{\frac{\overline{\neg \neg \varphi \vdash \neg \neg \varphi}^{ax}}{\varphi, \neg \varphi \vdash \perp} \neg_i}{\varphi \vdash \neg \neg \varphi} \rightarrow_i}{\vdash \varphi \rightarrow \neg \neg \varphi} \rightarrow_i \quad \frac{\vdash \neg \neg \varphi \rightarrow \varphi \quad \vdash \varphi \rightarrow \neg \neg \varphi}{\vdash \varphi \leftrightarrow \neg \neg \varphi} \wedge_i$$

$$5. \frac{\frac{\frac{\overline{\neg \psi \vdash \neg \psi}^{ax}}{\psi, \neg \psi \vdash \perp} \neg_i}{\psi \vdash \neg \neg \psi} \neg_i}{\varphi, \psi \vdash \neg \neg \varphi \wedge \neg \neg \psi} \wedge_i \quad \frac{\frac{\frac{\overline{\psi \vdash \psi}^{ax}}{\varphi, \neg \varphi \vdash \perp} \neg_i}{\varphi \vdash \neg \neg \varphi} \neg_i}{\varphi, \psi \vdash \neg \neg \varphi \wedge \neg \neg \psi} \wedge_i$$

Solution de l'exercice 2 :

1. Symétrie de l'égalité :

$$\text{avec } \phi(x) : x = x_1 \quad \frac{\vdash x_1 = x_1}{x_1 = x_2 \vdash x_1 = x_2} = i \quad \frac{\overline{x_1 = x_2 \vdash x_1 = x_2}^{ax}}{x_1 = x_2 \vdash x_2 = x_1} = e$$

$$\frac{x_1 = x_2 \vdash x_2 = x_1}{\vdash x_1 = x_2 \rightarrow x_2 = x_1} \rightarrow_i$$

$$\frac{\vdash x_1 = x_2 \rightarrow x_2 = x_1}{\vdash \forall x_2 (x_1 = x_2 \rightarrow x_2 = x_1)} \forall_i$$

$$\frac{\vdash \forall x_2 (x_1 = x_2 \rightarrow x_2 = x_1)}{\vdash \forall x_1 \forall x_2 (x_1 = x_2 \rightarrow x_2 = x_1)} \forall_i$$

2. Transitivité de l'égalité :

$$\text{avec } \phi(x) : x_1 = x \quad \frac{\overline{(x_1 = x_2 \wedge x_2 = x_3) \vdash (x_1 = x_2 \wedge x_2 = x_3)}^{ax}}{(x_1 = x_2 \wedge x_2 = x_3) \vdash x_2 = x_3} \wedge ed \quad \frac{\overline{(x_1 = x_2 \wedge x_2 = x_3) \vdash (x_1 = x_2 \wedge x_2 = x_3)}^{ax}}{(x_1 = x_2 \wedge x_2 = x_3) \vdash x_1 = x_2} \wedge eg$$

$$\frac{(x_1 = x_2 \wedge x_2 = x_3), (x_1 = x_2 \wedge x_2 = x_3) \vdash x_1 = x_3}{(x_1 = x_2 \wedge x_2 = x_3) \vdash x_1 = x_3} ctr$$

$$\frac{(x_1 = x_2 \wedge x_2 = x_3) \vdash x_1 = x_3}{\vdash ((x_1 = x_2 \wedge x_2 = x_3) \rightarrow x_1 = x_3)} \rightarrow_i$$

$$\frac{\vdash ((x_1 = x_2 \wedge x_2 = x_3) \rightarrow x_1 = x_3)}{\vdash \forall x_3 ((x_1 = x_2 \wedge x_2 = x_3) \rightarrow x_1 = x_3)} \forall_i$$

$$\frac{\vdash \forall x_3 ((x_1 = x_2 \wedge x_2 = x_3) \rightarrow x_1 = x_3)}{\vdash \forall x_2 \forall x_3 ((x_1 = x_2 \wedge x_2 = x_3) \rightarrow x_1 = x_3)} \forall_i$$

$$\frac{\vdash \forall x_2 \forall x_3 ((x_1 = x_2 \wedge x_2 = x_3) \rightarrow x_1 = x_3)}{\vdash \forall x_1 \forall x_2 \forall x_3 ((x_1 = x_2 \wedge x_2 = x_3) \rightarrow x_1 = x_3)} \forall_i$$

