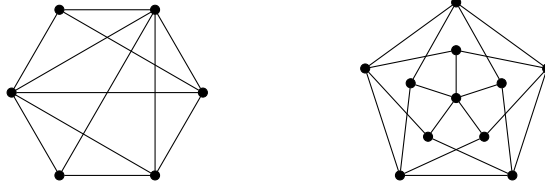


Graph Theory - Problem Set 5

October 10, 2024

Exercises

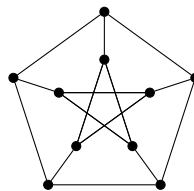
1. Determine if the following graphs are planar or not.



2. Determine all positive integers r and s for which $K_{r,s}$ is planar.
3. Let G be a graph on n vertices and $3n - 6 + k$ edges for some $k > 0$. Show that any drawing of G in the plane contains at least k crossing pairs of edges.
4. Let G be a planar graph with fewer than 12 vertices. Show that G has a vertex of degree at most 4.

Problems

5. Using Euler's formula, show that if G is a planar graph on n vertices that has finite girth g , then G has at most $\frac{g}{g-2}(n-2)$ edges. Deduce that the Petersen graph (see below) is not planar.



6. (a) Let G be a planar graph containing no triangles. Show that $\chi(G) \leq 4$.
(b) Let G be a planar graph containing at most three triangles. Show that $\chi(G) \leq 4$.
7. Prove that for any three vertices x, y, z of a planar graph on n vertices, the sum of the degrees $d(x) + d(y) + d(z)$ is at most $2n + 2$.
Hint: Use the fact that a planar graph does not contain $K_{3,3}$ as subgraph.
8. Let S be a set of n points in the plane such that any two of them have distance at least 1. Show that there are at most $3n - 6$ pairs of distance *exactly* 1.
Hint: Prove that the graph has no crossing using the triangle inequality.