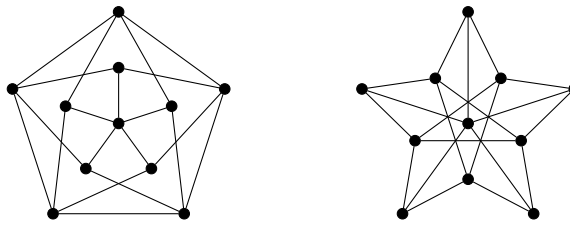


Graph Theory - Problem Set 3

September 26, 2024

Exercises

1. Show that a graph is connected if and only if it contains a spanning tree.
2. For what values of n does the graph K_n contain an Euler trail? An Euler tour? A Hamilton path? A Hamilton cycle?
3. (a) For what values of m and n does the complete bipartite graph $K_{m,n}$ contain an Euler tour?
(b) Determine the length of the longest path and the longest cycle in $K_{m,n}$, for all m, n .
4. (a) Find a graph such that every vertex has even degree but there is no Euler tour.
(b) Find a disconnected graph that has an Euler tour.
5. Determine the girth and circumference of the following graphs.



Problems

6. Let G be a connected graph that has an Euler tour. Prove or disprove the following statements.
 - (a) If G is bipartite then it has an even number of edges.
 - (b) If G has an even number of vertices then it has an even number of edges.
 - (c) For edges e and f sharing a vertex, G has an Euler tour in which e and f appear consecutively.
7. Show that if $k > 0$ then the edge set of any connected graph with $2k$ odd-degree vertices can be split into k trails. You have seen this problem in Problem Set 2 before. Here you are expected to find another solution.
Hint: Add some edges to get a multigraph which has Euler tour. Then split the tour.
8. Prove that in any connected graph G , there is a walk that uses each edge exactly twice.
9. Let G be a connected graph with an even number of edges such that all the degrees are even. Prove that we can color each of the edges of G red or blue in such a way that every vertex has the same number of red and blue edges touching it.