

Graph Theory - Problem Set 1

September 12, 2024

Exercises

1. Given a graph G with vertex set $V = \{v_1, \dots, v_n\}$ we define the *degree sequence* of G to be the list $d(v_1), \dots, d(v_n)$ of degrees in decreasing order. For each of the following lists, give an example of a graph with such a degree sequence or prove that no such graph exists:
 - (a) 3, 3, 2, 2, 2, 1
 - (b) 6, 6, 6, 4, 4, 3, 3
 - (c) 6, 6, 6, 4, 4, 2, 2
2. Construct two graphs that have the same degree sequence but are not isomorphic.
3. A graph is *k-regular* if every vertex has degree k . How do 1-regular graphs look like? And 2-regular graphs?
4. How many (labelled) graphs exist on a given set of n vertices? How many of them contain exactly m edges?

Problems

5. Prove that the number of odd-degree vertices in a graph is always even.
6. Let W be a closed walk that uses the edge e exactly once. Prove that W contains a cycle through e .
7. Show that every graph on at least two vertices contains two vertices of equal degree.
8. What is the maximum number of edges in a bipartite graph on n vertices? (Prove your answer.)
9. (*) Let G be a graph that contains a cycle C , and a path of length at least k between some two vertices of C (but they can also intersect elsewhere). Show that G contains a cycle of length at least $\sqrt{k}$.